Section 6.1: Fractions

- **Fraction:** A fraction is a number that can be represented by an ordered pair of whole numbers a and b as

$$\frac{a}{b} \quad \text{or} \quad a/b \quad \text{where } b \neq 0.$$

- **Proper Fraction:** A proper fraction is a fraction where the numerator is smaller in value than the denominator. A proper fraction always represents a quantity less than 1.

- **Improper Fraction:** An improper fraction is a fraction where the numerator is larger in value than the denominator.

- **Mixed Number:** A mixed numeral is a number of the form $a\frac{b}{c}$ where a, b, and c are nonzero whole numbers, and $b < c$.

  CAUTION: $a\frac{b}{c}$ means $a + \frac{b}{c}$ but the + symbol is not written. a is the whole number part and $\frac{b}{c}$ is the fraction part of the mixed number.

Example 1: Convert $3\frac{2}{5}$ to an improper fraction.

Example 2: Convert $\frac{28}{3}$ to mixed number.

- **Fundamental Law of Fractions:** The Fundamental Law of Fractions states that the value of a fraction does not change when its numerator and denominator are both multiplied by the same non-zero number. Thus, Let $\frac{a}{b}$ be any fraction and n a nonzero whole number. Then
$$\frac{a}{b} = \frac{an}{bn} = \frac{na}{nb}.$$

  NOTES:

  1. The fractions $\frac{a}{b}$ and $\frac{an}{bn}$ are called **equivalent fractions**.
  2. When $\frac{an}{bn}$ is replaced with $\frac{a}{b}$ where $n \neq 1$ we say $\frac{an}{bn}$ has been **simplified**. A fraction is written in **simplest form** (or **lowest terms**) when its numerator and denominator have no common factors.
  3. The Fundamental Law of Fractions can be used to add or subtract any two fractions.

- **Fraction Equality:** Let $\frac{a}{b}$ and $\frac{c}{d}$ be any fractions. Then
$$\frac{a}{b} = \frac{c}{d} \qquad \text{if and only if} \qquad ad = bc.$$

- **Less than for fractions:** Let $\frac{a}{c}$ and $\frac{b}{c}$ be any fractions. Then
$$\frac{a}{c} < \frac{b}{c} \qquad \text{if and only if} \qquad a < b.$$

- **Cross Multiplication of Fraction Inequality:** Let $\frac{a}{b}$ and $\frac{c}{d}$ be any fractions. Then
$$\frac{a}{b} < \frac{c}{d} \qquad \text{if and only if} \qquad ad < bc.$$

Example 3: Rewrite each of the following in simplest form.

(a) $\frac{189}{153}$

(b) $\frac{294}{63}$

(c) $\frac{480}{672}$

(d) $\frac{3335}{230}$

Example 4: Arrange the following fractions from smallest to largest: $\frac{4}{7}, \frac{7}{13}, \frac{14}{25}$

Example 5: Arrange the following fractions from smallest to largest: $\frac{5}{8}, \frac{6}{7}, \frac{3}{10}$

Example 6: Use pattern blocks to solve the following problems.

(a) The trapezoid is what fractional part of the hexagon?

(b) The blue rhombus is what fractional part of the hexagon?

(c) The triangle is what fractional part of the hexagon?

(d) The triangle is what fractional part of the blue rhombus?

(e) The triangle is what fractional part of the trapezoid?

Example 7: Use two trapezoids and one blue rhombus to construct a shape similar to the one shown below.

(a) Given that the shape = 1, what pattern block(s) would you use to represent each of the following fractions?

(i) $\frac{1}{4}$

(ii) $\frac{1}{2}$

(iii) $\frac{1}{8}$

(b) Given that the shape = 1, what fraction is represented by the yellow hexagon?