Section 6.3: Fraction Multiplication and Division

MULTIPLICATION OF FRACTIONS

- **Repeated Addition:** Multiplication can be viewed as repeated addition.

$$5 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$$

  * Problem: What if we have $\frac{1}{5} \times \frac{2}{3}$? Now we cannot use repeated addition.

- **Rectangular Array:**

- **Multiplication of Fractions:** Let $\frac{a}{b}$ and $\frac{c}{d}$ be any fractions. Then $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$.

  * Important: $2\frac{1}{5} = 2 + \frac{1}{5}$.

Example 1: Calculate the following and express as a proper fraction or mixed number in simplest form.

(a) $\frac{2}{3} \times \frac{15}{4} =$

(b) $\frac{16}{5} \times \frac{35}{24} =$

(c) $8\frac{1}{4} \times 3\frac{4}{5} =$

(d) $\frac{3}{5} + \frac{2}{3} \times \frac{4}{7} =$

Properties of Fraction Multiplication

- **Closure Property:** The product of two fractions is a fraction.
- **Commutative Property:** $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$.
- **Associative Property:** $\frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f}$
- **Multiplicative Identity Property:** One is the multiplicative identity. Therefore, $\frac{a}{b} \times 1 = \frac{a}{b} = 1 \times \frac{a}{b}$.
- **Multiplicative Inverse Property:** For every nonzero fraction $\frac{a}{b}$, there is an unique fraction $\frac{b}{a}$ such that $\frac{a}{b} \times \frac{b}{a} = 1$.
  - For $\frac{a}{b} \neq 0$, $\frac{b}{a}$ is called the **multiplicative inverse** or **reciprocal** of $\frac{a}{b}$.
- **Distributive Property:** $\frac{a}{b}\left(\frac{c}{d} + \frac{e}{f}\right) = \frac{ac}{bd} + \frac{ae}{bf}$.

Example 2: Solve for x: $\frac{2}{3}x = \frac{3}{5}$

DIVISION OF FRACTIONS:

- **Visual Approach:** When we want to solve $\frac{15}{16} \div \frac{3}{16}$, we are asking how many $\frac{3}{16}$ are there in $\frac{15}{16}$.

- As a result of the missing factor approach, we have

$$\frac{35}{16} \div \frac{7}{8} = \frac{35 \div 7}{16 \div 8} = \frac{5}{2}$$

- **Division of Fractions:** Let $\frac{a}{b}$ and $\frac{c}{d}$ be any fractions with $c \neq 0$. Then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.

- **Why do we invert and multiply when we divide fractions?**
  - **Multiplication Approach:**
  - **Missing Factor Approach:**

Example 3: Calculate the following and express the answer as a proper fraction or mixed number in simplest form.

(a) $\frac{8}{11} \div \frac{4}{3}$

(b) $\frac{33}{14} \div \frac{11}{7}$

(c) $4\frac{3}{4} \div 3\frac{5}{8}$

(d) $\frac{1}{3} + \frac{5}{4}\left(\frac{1}{4} \div \frac{5}{6}\right)$

Example 4: Inga was making a cake that called for 4 cups of flour. However, she could only find a two-thirds measuring cup. How many two-thirds measuring cups of flour will she need to make her cake?

Example 5: An auditorium contains 315 occupied seats and was $\frac{7}{9}$ filled. How many seats are in the auditorium?

Example 6: Mary bought $20\frac{1}{2}$ yards of material to make aprons. Each apron requires $1\frac{2}{5}$ yards of material.

(a) How many aprons can Mary make?

(b) How much fabric is left over? Exact answer only.

Example 7: Kids belonging to the Boys and Girls Club of America collected cans and bottles to raise money by returning them for the deposit. If 54 more cans than bottles were collected and the number of bottles was $\frac{5}{11}$ of the total number of beverage containers collected, how many bottles and how many cans were collected?

Example 8: Ticket sales for this season of the Performing Arts Center have been brisk. Two-ninths of the people purchased 5 or more tickets, one-third purchased 4 tickets, one-fourth purchased 3 tickets, one-eighth purchase 2 tickets and 360 people purchased a single ticket. How many people have purchased tickets for this season of the Performing Arts Center?

Example 9: Upon his death, Mr. Freespender left $\frac{1}{2}$ of his estate to his wife, $\frac{1}{8}$ to each of his two children, $\frac{1}{16}$ to each of his three grandchildren, and the remaining $\$15,000$ to his favorite university. What was the value of his entire estate?