Section 8.1: Addition and Subtraction of Integers

- **Integers:** The set of integers is the set $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. The numbers $1, 2, 3, \ldots$ are called positive integers and the numbers $-1, -2, -3, \ldots$ are called negative integers. Zero is neither a positive nor negative integer.

- **Representing integers using a set model:** In the set model, we can use $+$ to represent positives and $-$ to represent negatives.

- **Integer Number Line:**

- **Opposite:** The opposite of an integer a, written $-a$ or $-(a)$, is defined as
  - represented by the same number of symbols as a but of the opposite symbol in the set model.
  - represented by the integer that is the mirror image about zero on the number line.

- **Absolute Value:** The absolute value of an integer a, written $|a|$, is defined to be the distance from a to zero on the integer number line. As a result, $|a|$ is always a non-negative number.

ADDITION:

- **Set Model:**

- **Number Line (Measurement) Model:**

INTEGER ADDITION FACTS:

- positive + positive = positive
- negative + negative = negative
- positive + negative = cannot be determined
- negative + positive = cannot be determined

Properties of Integer Addition

- Closure Property: integer + integer = integer.
- Commutative Property: If a and b are integers, then $a + b = b + a$.
- Associative Property: If a, b, and c are integers, then $a + (b + c) = (a + b) + c$.
- Identity Property: Zero is the unique integer such that $a + 0 = a = 0 + a$ for all integers a. We say that 0 is the additive identity.
- Additive Inverse Property: For each integer a, there is a unique integer $-a$, such that $a + (-a) = 0 = (-a) + a$. We say that $-a$ is the additive inverse, or opposite, of a. NOTE: $-a$ is not necessarily negative.

NOTE: The additive inverse property is a property that integer addition has that whole number addition did not.

SUBTRACTION:

- **Take-away approach:**

- **Adding the opposite:** Let a and b be any integers. Then $a - b = a + (-b)$.

- **Missing addend approach:** Let a, b, and c be any integers. Then $a - b = c$ if and only if $b + c = a$.

Property of Integer Subtraction

- Closure Property: integer − integer = integer.