Section 8.2: Multiplication and Division of Integers

MULTIPLICATION:

- **Number line:** Recall that the multiplication of whole numbers can be viewed as repeated addition.

- **Pattern:**

- **Charge Field:** $a \times b$

  1. Begin with a set of zero.
  2. If $a > 0$, then we add a groups of b to our set.
  3. If $a < 0$, then we take away $|a|$ groups of b from our set.

Example 1: Illustrate 3×-4 using the charge field method.

Example 2: Illustrate -5×-2 using the charge field method.

INTEGER MULTIPLICATION FACTS:

- $a \times 0 = 0 = 0 \times a$
- positive $\times$ negative = negative
- positive $\times$ positive = positive
- negative $\times$ negative = positive

Properties of Integer Multiplication

- Closure Property: integer $\times$ integer = integer.
- Commutative Property: If a and b are integers, then $a \cdot b = b \cdot a$.
- Associative Property: If a, b, and c are integers, then $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.
- Identity Property: One is the unique number such that $a \cdot 1 = a = 1 \cdot a$ for all integers a. We say that 1 is the multiplicative identity.
- Distributive Property: If a, b, and c are integers, then $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.
- Multiplication Cancellation Property: Suppose $c \neq 0$. If $ac = bc$ then $a = b$.
- Zero Divisors Property: $ab = 0$ if and only if $a = 0$ or $b = 0$.

DIVISION: Let a and b be integers with $b \neq 0$. Then $a \div b = c$ if and only if $a = b \cdot c$ for a unique integer c. (Recall that this is the missing factor approach).

INTEGER DIVISION FACTS:

- $a \div 1 = a$
- positive $\div$ negative $=$ negative
- positive $\div$ positive $=$ positive
- negative $\div$ negative $=$ positive
- negative $\div$ positive $=$ negative
- If $a \neq 0$, then $0 \div a = 0$
- $a \div 0 =$ undefined
- $0 \div 0 =$ undefined

Example 3: Let a be a negative integer, b be a positive integer, and c be a negative integer. Determine if each of the following is positive, negative, or cannot be determined.

(a) $(a-b)(b-c)$

(b) $4a - 3b + 9c$

(c) $(a+c)(b+c)$

(d) $a + bc$

NEGATIVE EXPONENTS: Let a be any nonzero number and n be a positive integer. Then

$$a^{-n} = \frac{1}{a^n} \qquad \text{and} \qquad \frac{1}{a^{-n}} = a^n.$$

NOTE: All of the exponent rules learned in Section 3.3 hold for negative exponents.

Example 4: Simplify each of the following.

(a) $(-5)^2 =$

(b) $-6^2 =$

(c) $4^{-2} =$

(d) $5^{-3} =$

(e) $(-2)^{-4} =$

(f) $-7^{-2} =$

(g) $(-5)^3 =$

(h) $-2^{-5} =$

Scientific Notation: Numbers are said to be in scientific notation when expressed in the form $a \times 10^n$, where $1 \leq |a| < 10$ and n is any integer. The number a is called the **mantissa** and n the **characteristic** of $a \times 10^n$.

Converting from Standard From to Scientific Notation:

Step 1: Write the number factor a by moving the decimal point so that $|a|$ is greater than or equal to 1 but less than 10. To do this, place the decimal point to the right of the first non-zero digit.

Step 2: Multiply the number by 10^n, where $|n|$ is the number of places that the decimal point moves. If the decimal point moves to the left, then $n > 0$ (n is positive). If the decimal point moves to the right, then $n < 0$ (n is negative). Remove any zeros lying to the right of the last non-zero digit or to the left of the first non-zero digit.

Example 5: Write each number in scientific notation.

(a) $340,000,000$

(b) 0.0000000467

(c) -0.0000901200

(d) $56,800,000,000,000,000$

Converting from Scientific Notation to Standard Form:

Step 1: Remove the exponential factor 10^n.

Step 2: Move the decimal point $|n|$ places, inserting zero placeholders as needed. If $n > 0$ (n is positive), move the decimal point to the right. If $n < 0$ (n is negative), move the decimal point to the left.

Example 6: Write each number in standard form.

(a) 4.98×10^{-5}

(b) 9.4×10^7

(c) -3.129×10^9

(d) 1.234×10^{-6}

Example 7: Evaluate each of the following. Express your answers in scientific notation.

(a) $\left(9.62 \times 10^{-12}\right)\left(2.8 \times 10^{9}\right)$

(b) $\dfrac{3.74 \times 10^{-6}}{8.5 \times 10^{-15}}$

(c) $\dfrac{\left(1.38 \times 10^{12}\right)\left(4.5 \times 10^{-16}\right)}{1.15 \times 10^{10}}$