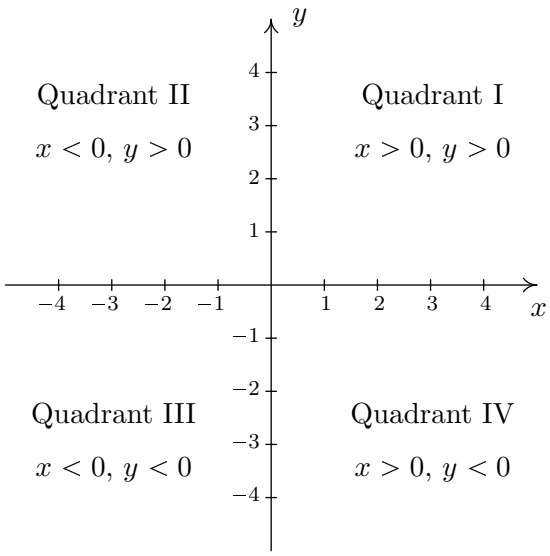
Section 9.3: Functions and their Graphs

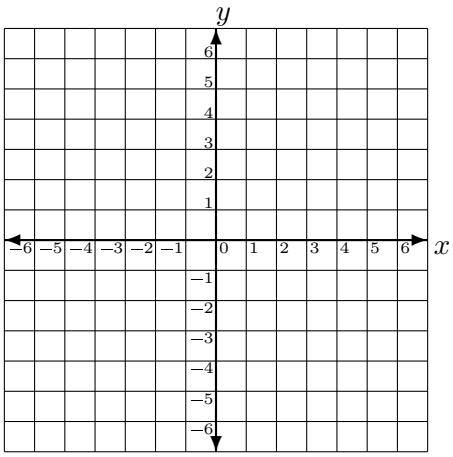
Graphs provide a way of displaying, interpreting, and analyzing data in a visual format. In many problems, we will consider two variables. Therefore, we will need to have two axes – one for the x variable and another for the y variable. Together these axes will form the **Rectangular Coordinate System**, or **Cartesian Coordinate System**. The horizontal axis is the $x-$axis and the vertical axis is the $y-$axis. These two axes divide the $xy-$plane into four **quadrants** and the intersection of the two axes is called the **origin**. See the following diagram.



Ordered pair: Each point in the plane is called an **ordered pair** and is denoted (x, y). The first number x indicates the point's horizontal location with respect to the y-axis, and the second number y indicates the point's vertical location with respect to the x-axis. Hence, the origin is labeled $(0, 0)$.

Example 1: Plot the following points on the same set of axes:

$$A = (2, -3),\ B = (-2, 3),\ C = (-1, -4),\ \text{and } D = (1, 5).$$



- The x-**intercept** of a graph is the point where the graph crosses the x-axis. This point is $(a, 0)$ where to find a, we let $y = 0$ and solve for x.
- The y-**intercept** of a graph is the point where the graph crosses the y-axis. This point is $(0, b)$ where to find b, we let $x = 0$ and solve for y.

Example 2: Find the x-intercept and y-intercept for $7x - 3y = 2$.

- **Function:** A function is a rule or correspondence that assigns to each element of one set, called the domain, exactly one element of a second set, called the range. A function may be defined by a set of ordered pairs, a table, a graph, or an equation.
- **Domain:** The domain of a function is the set of all inputs. If x is any element in the domain, then x is called the **independent variable**.
- **Range:** The range of a function is the set of all outputs. If y represents an output of the function f from an input x, then y is called the **dependent variable** and is denoted by $f(x)$.

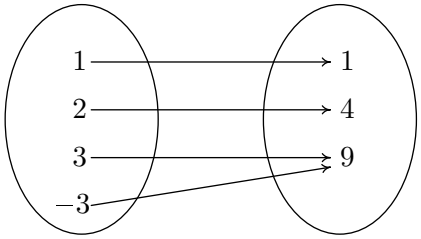
Example 3: Determine which of the following are examples of functions. For each function, determine the domain and range.

(a) $\{(1,2),\ (3,6),\ (6,8),\ (9,2),\ (12,5)\}$

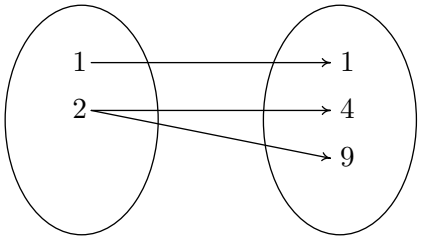
(b)

x	1	2	-5	2	-4
y	-1	4	6	7	9

(c)



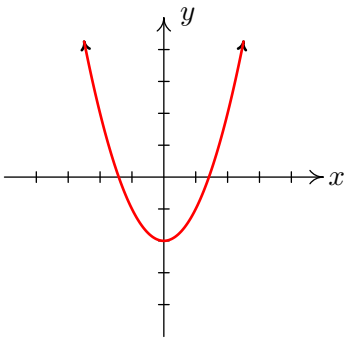
(d)

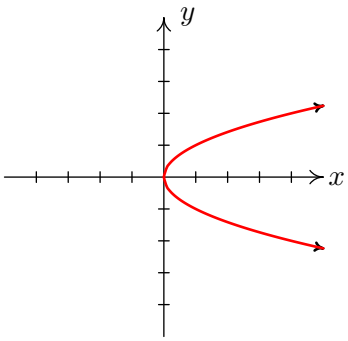


- The **graph of a function** is a set of points (x, y) in the xy-plane such that $y = f(x)$.

- **The Vertical Line Test:** A set of points in the xy-plane is the graph of a function if and only if no vertical line intersects the set of points more than once.

Example 4: Determine if each of following curves is the graph of a function.



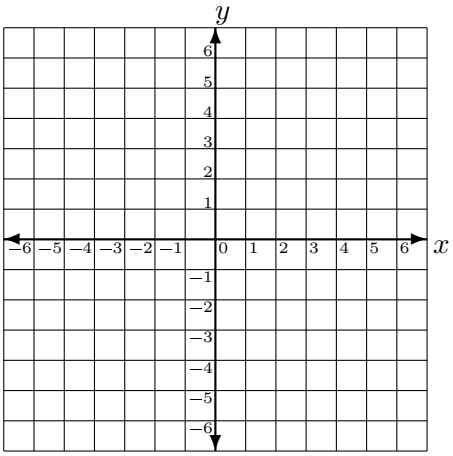


Linear Functions: Linear functions are functions whose graphs are lines. A linear function has the algebraic form

$$f(x) = mx + b$$

where m and b are constants. In the function $f(x) = 3x + 2$, $m = 3$ and $b = 2$.

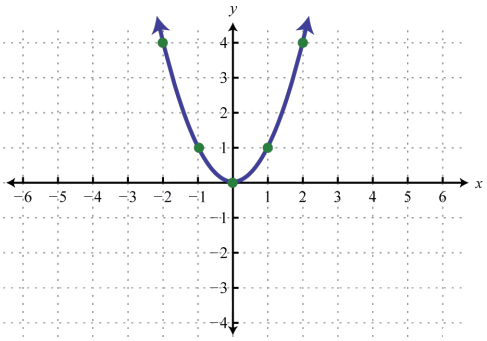
Example 5: Make a table of at least five values for $f(x) = 3x + 2$ and sketch the graph.

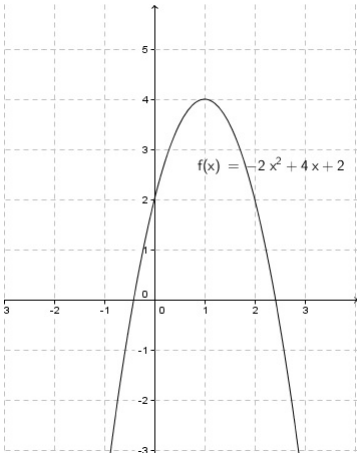


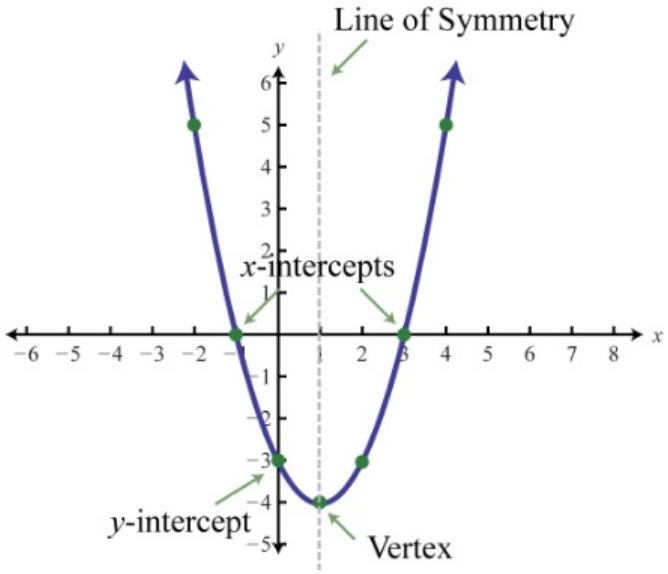
Quadratic functions: A quadratic function is a function of the form $f(x) = ax^2 + bx + c$, where a, b, and c are constants and $a \neq 0$. The graph of a quadratic function is a **parabola**. Below of some graphs for quadratic functions.

$f(x) = x^2$

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4

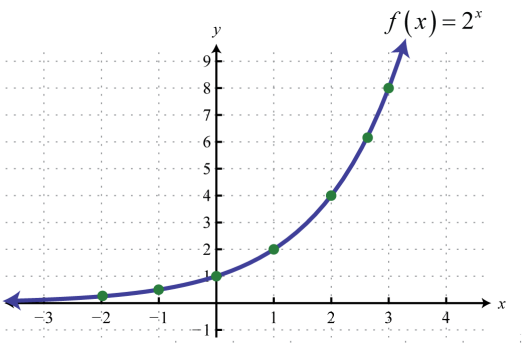


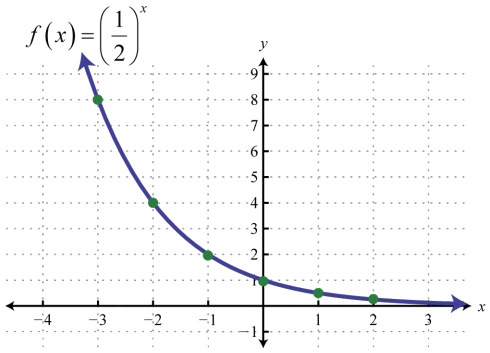




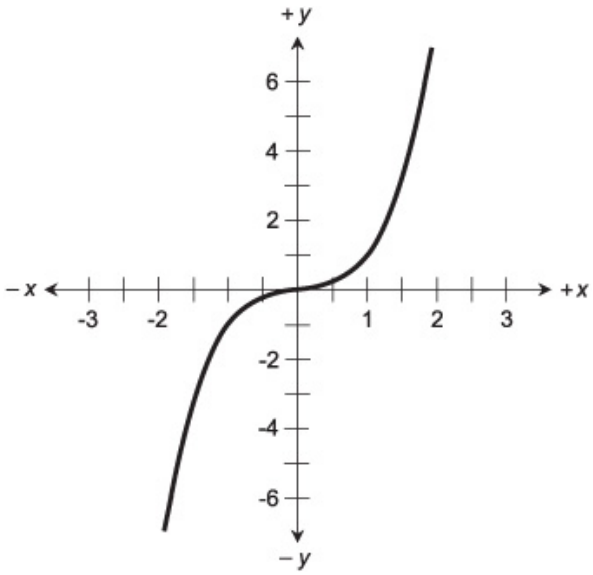
NOTE: When $a > 0$, the parabola opens up. When $a < 0$, the parabola opens down.

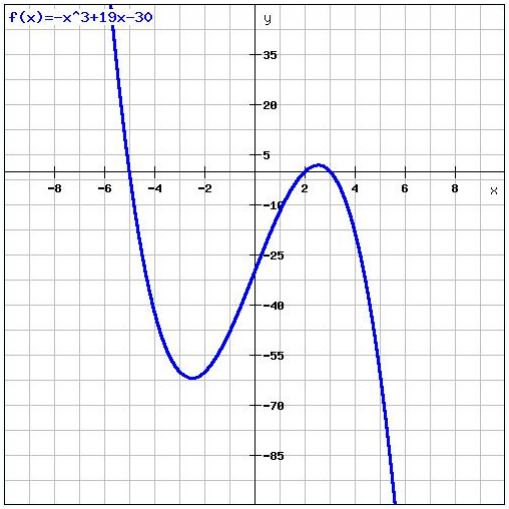
Exponential functions: The function $f(x) = a^x$, where x is a real number, $a > 0$ and $a \neq 1$, is called an **exponential function** with base a. If $a > 1$, then the function is increasing; if $0 < a < 1$, then the function is decreasing. Below are two graphs of exponential functions.



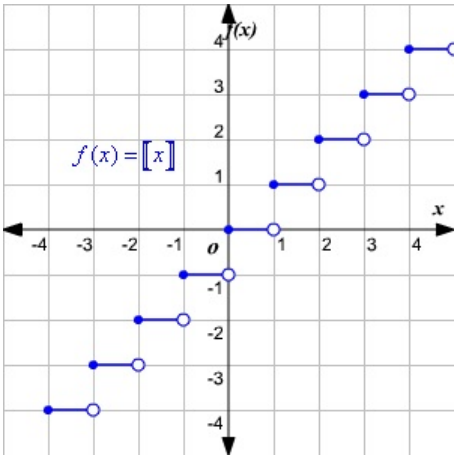


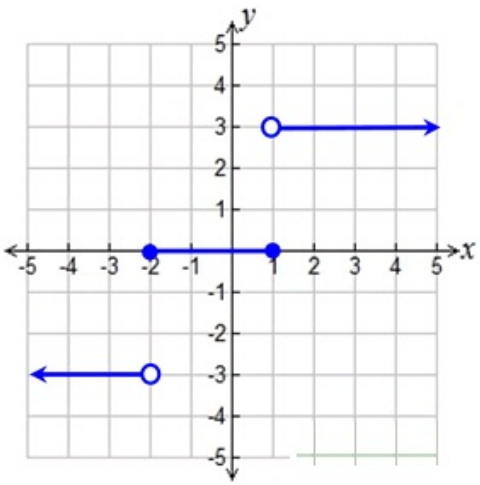
Cubic functions: A cubic function is a function of the form $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c, and d are constants and $a \neq 0$. Below are two graphs of cubic functions.





Step functions: A step function is a function that increases or decreases from one constant value to another. One of the most common step functions is the **greatest integer function** $f(x) = [\![x]\!]$ which is defined to be the greatest integer that is less than or equal to x. For example, $[\![3.8]\!] = 3$ and $[\![-2.1]\!] = -3$. Below are two graphs of step functions.

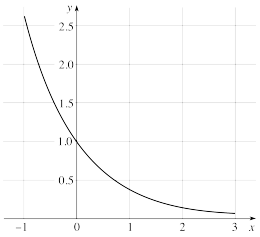


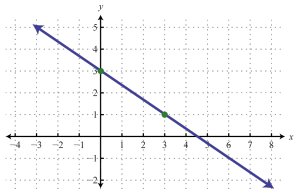


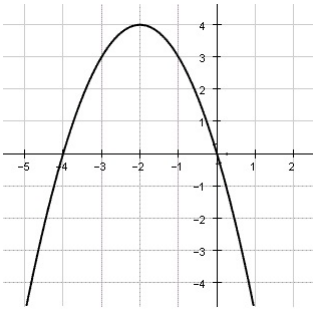
Example 6: Find the following.

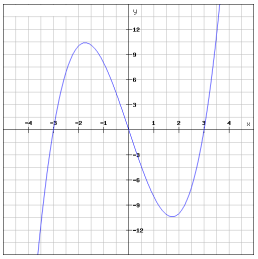
(a) $[\![-4.2]\!] =$ (b) $[\![8.5]\!] =$ (c) $\left[\!\left[\frac{14}{3}\right]\!\right] =$

Example 7: Determine which type of function best fits each of the following graphs: linear, quadratic, exponential, cubic, or step?

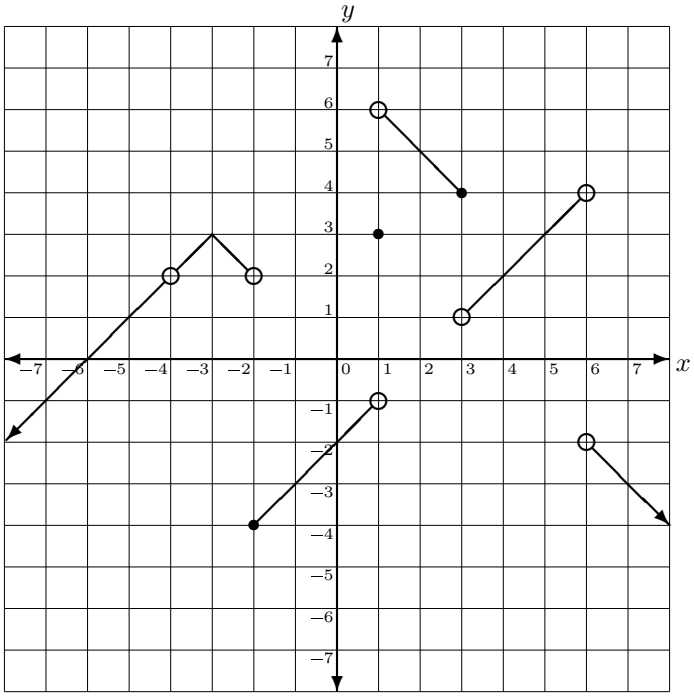








Example 8: For the function f graphed below, find the following:



(a) $f(-5)$

(b) $f(-4) =$

(c) $f(1) =$

(d) $f(3) =$

(e) $f(5) =$

(f) $f(6) =$