Section 3.2: Whole Numbers Multiplication & Division

MULTIPLICATION: factor · factor = product

- **Repeated Addition Approach**: Let a and b be any whole numbers where $a \neq 0$. Then
$$a \cdot b = \underbrace{b + b + \cdots + b}_{a \text{ times}}$$

- **Rectangular Array Approach**: Let a and b be any whole numbers. Then $a \cdot b$ is the number of elements in a rectangular array having a rows and b columns.

- **Cartesian Product Approach**: Let a and b be any whole numbers. If $n(A) = a$ and $n(B) = b$, then $a \cdot b = n(A \times B)$.

PROPERTIES OF WHOLE NUMBER MULTIPLICATION

- **Closure Property:** The product of any two whole numbers is a whole number.

  Example 1: Determine if the following sets are closed under multiplication.

  (a) $\{0, 1\}$

  (b) $\{0, 1, 2\}$

- **Commutative Property:** Let a and b be whole numbers. Then

$$a \cdot b = b \cdot a.$$

- **Associative Property:** Let a, b, and c be any whole numbers. Then

$$(a \cdot b) \cdot c = a \cdot (b \cdot c).$$

- **Identity Property:** There is a unique whole number 1 such that for all whole numbers a,

$$a \cdot 1 = a = 1 \cdot a.$$

  One is called the **multiplicative identity**.

- **Distributive Property:** Let a, b, and c be whole numbers. Then

$$a(b + c) = ab + ac$$
$$a(b - c) = ab - ac$$

- **Multiplication Property of Zero:** For every whole number a,

$$a \cdot 0 = 0 \cdot a = 0.$$

DIVISION: dividend $\div$ divisor $=$ quotient

- **Repeated Subtraction Approach:**

- **Missing Factor Approach:** If a and b are any whole numbers with $b \neq 0$, then $a \div b = c$ if and only if $a = b \cdot c$ for some whole number c.

- **Division Algorithm:** If a and b are any whole numbers with $b \neq 0$, then there exist unique whole numbers q and r such that

$$a = bq + r,$$

where $0 \leq r < b$. (Here b is called the divisor, q is called the quotient, and r is the remainder.

- **Division by and with Zero:**
  1. If $a \neq 0$, then $0 \div a = 0$
  2. If $a \neq 0$, then $a \div 0 =$ undefined.
  3. $0 \div 0 =$ undefined.