Topic 1: Arguments

An **argument** is a set of statements in which one of the statements is called the conclusion and the rest comprise the hypotheses.

A **valid argument** is an argument in which the conclusion must be true whenever the hypothesis is true.

If it is snowing, then it is cold.
It is snowing.

Therefore, it is cold.

Let p = it is snowing and let q = it is cold. Then symbolically, the above argument can be represented as:

$$\begin{array}{l} p \to q \\ p \\ \hline q \end{array}$$

We can determine the validity of an argument by considering its corresponding truth table.

p	q	$p \to q$	$(p \to q) \wedge p$	$[(p \to q) \wedge p] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

NOTES:

- We always connect all hypotheses with a $\wedge$.
- The argument is valid if and only if all the truth values in the last column are true.

An **invalid argument** is an argument in which the conclusion can be false when its hypothesis is true.

If it is raining, then the streets are wet.
The streets are wet.

Therefore, it is raining.

Let p = it is raining and let q = the streets are wet. Then symbolically, the above argument can be represented as:

$$\begin{array}{l} p \to q \\ \underline{q} \\ p \end{array}$$

Again, we determine the validity of this argument by considering its corresponding truth table:

p	q	$p \to q$	$(p \to q) \wedge q$	$[(p \to q) \wedge q] \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

NOTE: The argument is invalid because not all of the truth values in the last column are true.

Examples: Use truth tables to determine if the following arguments are valid or invalid.

1. $$\begin{array}{l} q \wedge \sim p \\ \underline{\sim p} \\ q \end{array}$$

2.

If you like Tums, then you'll like Rolaids.
You don't like Rolaids.

You don't like Tums.

3.

$\sim p \rightarrow q$
$q \rightarrow p$
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q

4.

$$\frac{\begin{array}{c} p \wedge \sim q \\ \sim p \longrightarrow q \end{array}}{\sim (p \vee q)}$$

5.

$$\frac{\begin{array}{c} \sim p \vee q \\ p \longleftrightarrow (q \wedge \sim p) \end{array}}{q \wedge p}$$