Section 3.3: Exponents and Order of Operations

Definition:

- An **whole number exponent** is a number that tells how many times a factor is repeated in a product. For example, in the problem 2^4, 2 is called the base and 4 is the exponent.

$$2^4 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{\text{4 times}} = 16.$$

Exponent Rules:

- **Product Rule:** For any whole numbers m and n,

$$\boxed{a^m \cdot a^n = a^{m+n}.}$$

When multiplying like bases, we add the exponents.

- **Quotient Rule:** For any nonzero number a and any whole numbers m and n,

$$\boxed{\frac{a^m}{a^n} = a^{m-n}.}$$

When we divide like bases, we subtract the exponents.

- **Power Rule:** For any whole numbers m and n,

$$\boxed{(a^m)^n = a^{mn}.}$$

When we raise a power to another power, we multiply the exponents.

- For any whole number m,

$$\boxed{(ab)^m = a^m \cdot b^m.}$$

When we have a product raised to a power, we raise each factor to the power.

- For any whole number m,

$$\boxed{\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}.}$$

When we have a quotient raised to a power, we raise both the numerator and denominator to the power.

- **Zero Exponent Rule:** For any nonzero real number a,

$$\boxed{a^0 = 1.}$$

Common Mistakes to Avoid:

- When using the product rule, the bases MUST be the same. If they are not, then the expressions cannot be combined. Also, remember to keep the bases the same and only add the exponents. For example,

$$3^2 \cdot 3^4 = 3^{2+4} = 3^6 \qquad\qquad 3^2 \cdot 3^4 \neq 9^6.$$

- When using the quotient rule, the bases MUST be the same. If they are not, then the expressions cannot be combined. Also, remember to keep the bases the same and only subtract the exponents. For example,

$$\frac{4^5}{4^3} = 4^2 = 16.$$

- The Power Rule and Quotient Rule do NOT hold for sums and differences. In other words,

$$(a+b)^m \neq a^m + b^m \qquad \text{and} \qquad (a-b)^m \neq a^m - b^m.$$

Example 1: Rewrite the following using a single exponent.

(a) $x^{15} \div x^3$

(b) $y \cdot 2 \cdot x \cdot y \cdot x \cdot x$

(c) $25^9 \div 5^4$

(d) $8^6 \cdot \left(2^4 \cdot 2^3\right)^3$

(e) $16^7 \cdot 4^8 \cdot 8^3 \div 2^{12}$

(f) $2^4 \cdot 32^2 \div \left(8^2 \cdot 4^3\right)$

(g) $27^6 \div 3^8 \cdot 9^4 \cdot 81$

(h) $125^2 \cdot 9^4 \cdot 27^2 \cdot 25^4$

Example 2: Using properties of exponents, determine the larger of the following pairs. Explain your reasoning.

(a) 4^{14} or 8^{10}

(b) 6^{10} or 3^{20}

(c) 8^{23} or $8^{22} + 8^{22} + 8^{22} + 8^{22}$

Example 3: Find x.

(a) $5^2 \cdot 5^x = 5^{19}$

(b) $49^x \cdot 7^5 = \left(7^3\right)^5$

Order of Operations: In mathematics the **order of operations** is a collection of rules that define which procedures to perform first in order to evaluate a given mathematical expression. A mnemonic which may be useful when memorizing the order of operations is **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally. The first letters of the words of the mnemonic represent:

Order of Operations

When more than one operation is present in an algebraic expression, the proper order for doing the operations is:

1. **P**arentheses - or any grouping symbol
2. **E**xponents
3. **M**ultiplication and **D**ivision, left to right in order of appearance
4. **A**ddition and **S**ubtraction, left to right in order of appearance

Example 4: Evaluate each of the following expressions.

(a) $18 + 6 \cdot 12 - 4$

(c) $6 + 24 \div 3 + 7$

(b) $12 + 16 \cdot 2 + 9 \cdot 8$

(d) $3 \cdot 8 \div 2 \cdot 4$

(e) $5 \cdot (10 - 4) - 3^2$

(f) $(50 + 10) \div 5 - 2^2$

(g) $50 + 10 \div 5 - 2^2$

(h) $27 \div 3(6 - 3)^2$

(i) $(16 + 14) \div 2 \cdot 5$

(j) $(9 - 4)^2 \cdot (2 + 4)$