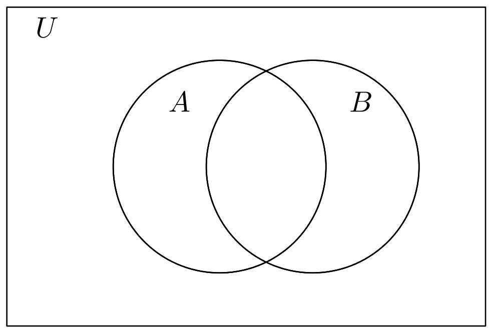
Section 2.1:   Set Operations

Venn Diagrams: are diagrams used to represent the relationship between sets. (U is the universal set and it includes all items under discussion at a given time.)



Example 1: Draw a Venn Diagram to represent the following relationships between set A and set B.

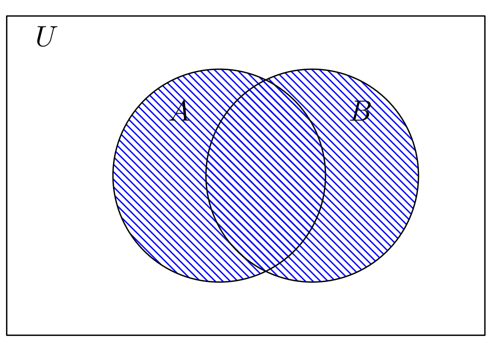
(a) $B \subset A$

(b) A and B are disjoint sets.

SET OPERATIONS:

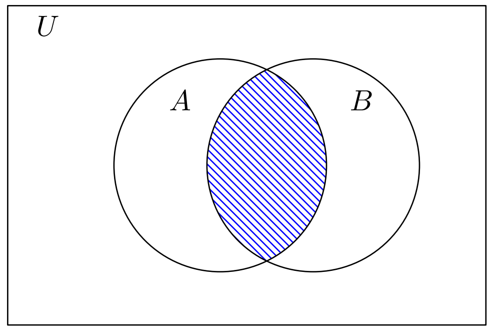
- **Union of sets:** The union of two sets A and B, denoted $A \cup B$, is the set that consists of all elements belonging either to A or to B (or both). That is,

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$



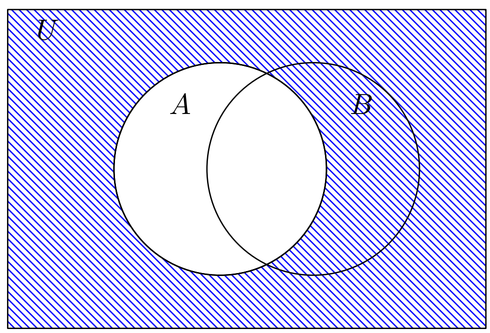
- **Intersection of sets:** The intersection of sets A and B, denoted $A \cap B$, is the set of all elements common to sets A and B. That is,

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$



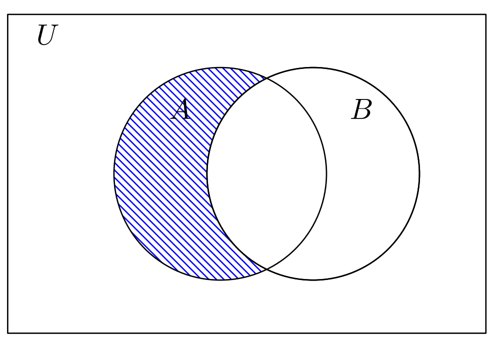
- **Complement of a set:** The complement of a set, denoted $\overline{A}$, is the set of all elements in the universe U that are not in A. That is,

$$\overline{A} = \{x \mid x \in U \text{ and } x \notin A\}$$



- **Difference of sets:** The set difference of set B from set A, denoted $A - B$, is the set of all elements in A that are not in B. That is,

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$



- **Cartesian Product:** The cartesian product of set A with set B, denoted $A \times B$ and read A cross B, is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$.

Example 2: Given the following sets:

$$\begin{aligned} U &= \{1,2,3,4,5,6,7,8\} \\ A &= \{1,2,3,4,5\} \\ B &= \{1,2,5,7\} \\ C &= \{5,6,7\} \end{aligned}$$

Find each of the following:

(a) $A \cup C$

(b) $B \cap A$

(c) $\overline{B}$

(d) $\overline{A}$

(e) $C - A$

(f) $\overline{B \cup C}$

(g) $\overline{B} \cap \overline{C}$

(h) $C \times B$

(i) $(A \cap C) \cup \overline{B}$

(j) $C \cap \overline{(B \cup A)}$

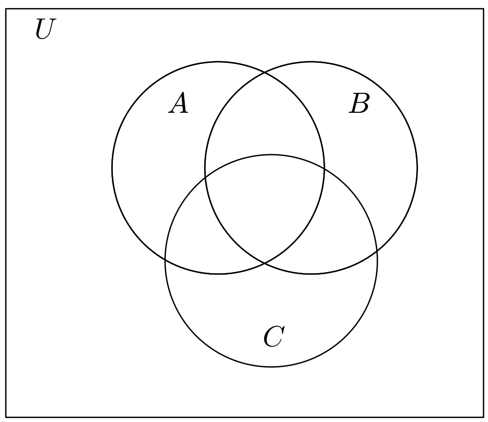
- **DeMorgan's Laws for Sets:** For all sets A and B, we have

$$\overline{A \cup B} = \overline{A} \cap \overline{B}. \qquad \overline{A \cap B} = \overline{A} \cup \overline{B}.$$

Example 3: Given the following sets:

$$\begin{aligned} U &= \{a, b, c, d, e, f, g, h, i, j\} \\ A &= \{b, c, d, e, g, h\} \\ B &= \{d, g, i, j\} \\ C &= \{a, d, h, i\} \end{aligned}$$

Place the elements of these sets in their proper locations on the following Venn Diagram.

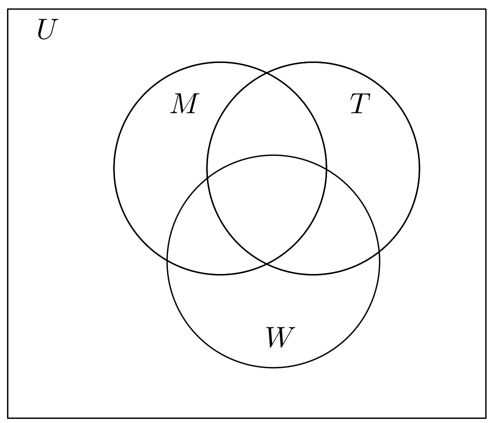


Example 4: A university professor asked his class of 42 students when they had studied for his exam last week. Their responses were as follows:

9	students had studied on Monday (M)
18	students had studied on Tuesday (T)
30	students had studied on Wednesday (W)
3	students had studied both Monday and Tuesday
10	students had studied both Tuesday and Wednesday
6	students had studied both Monday and Wednesday
2	students has studied on Monday, Tuesday, and Wednesday

Assuming all 42 students responded and answered honestly, answer the following questions.

(a) Fill in the following Venn Diagram COMPLETELY using the data given above.



(b) How many students studied on Wednesday but not on either Monday or Tuesday?

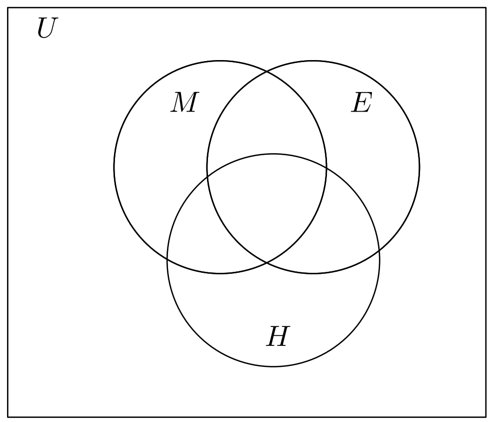
(c) How may student did all of their studying on one day?

(d) How many students did not study at all for his exam last week?

Example 5: A survey of 100 randomly selected students gave the following information:

45	students are taking Mathematics (M)
41	students are taking English (E)
40	students are taking History (H)
15	students are taking Math and English
18	students are taking Math and History
17	students are taking English and History
7	students are taking all three (Math, English and History)

(a) Fill in the following Venn Diagram COMPLETELY using the data given above.



(b) How many students are taking only Mathematics?

(c) How many students are taking Mathematics or English?

(d) How many students are taking History but not English?

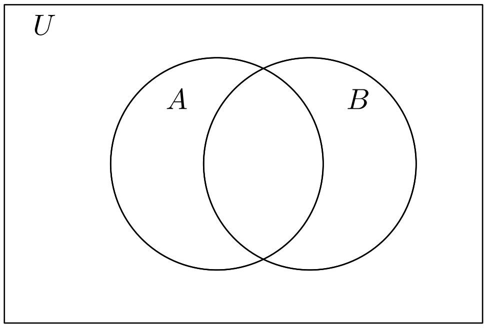
(e) How many students are NOT taking any of these courses?

(f) How many students are taking English and Mathematics, but not History?

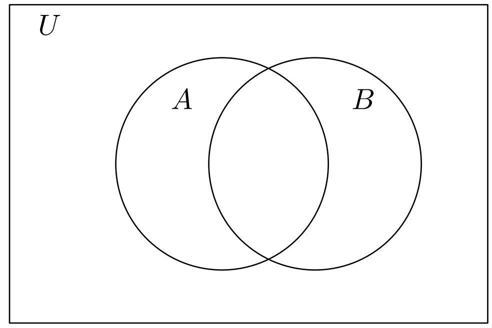
(g) How many students are taking History or English, but not Mathematics?

Example 6: In each Venn Diagram, shade the area corresponding to the designated set.

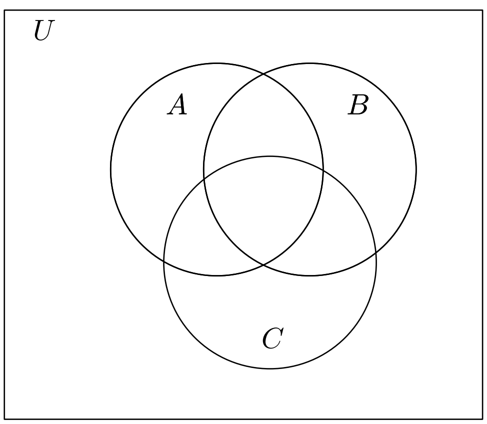
(a) $A \cap \overline{B}$



(b) $\overline{A} \cup \overline{B}$



(c) $\overline{(A \cap B)} \cap C$



(d) $(A \cup B) \cap \overline{C}$