Sections 1.1/1.2:   Problem Solving

Four Phases of Problem Solving

George Polya's four phases of problem solving have become the framework often recommended for teaching problem solving. The four phases are listed below.

1. Understand the problem.

2. Devise a plan to solve the problem.

3. Carry out the plan.

4. Look back to examine your solution.

Some Problem Solving Strategies

- Act it out.

- Draw a picture or diagram.

- Construct a table or chart.

- Make a list.

- Find a pattern.

- Solve a simpler problem.

- Guess and check.

- Working backwards.

- Consider all possibilities.

1. Place the numbers $1, 2, 3, 4, 5, 6, 7$ into the circles so that any three numbers in a line through the center give the same sum.
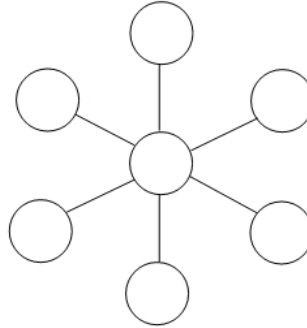
2. A farmer looks out in the barnyard and sees a number of pigs and chickens. The farmer says to her daughter, "I count 24 heads and 80 feet." How many pigs and how many chickens are out there?

3. Find the missing term in the following sequence.

$$10, \ 17, \ \_\_, \ 37, \ 50, \ 65, \ 82, \ 101$$

4. Mike said to Linda, "Bet you can't guess the number of candies I have in this sack."

   "Give me a clue," she said.

   "I have more than 50 but fewer than 125. If you divide them into piles of 8, there are 2 left over. If you divide them into piles of 7, there is 1 left over," said Mike.

   How many candies does Mike have?

5. Find the number below that is described by the following clues.

   - The sum of the digits is 14.
   - The number is a multiple of 5.
   - The number is in the thousands.
   - The number is not odd.
   - The number is less than 2411.

2570	2290	2660	1355	1832
1058	2435	1580	1455	860
1922	1770	905	770	1680

6. In a certain bank the positions of cashier, manager, and teller are held by Brown, Jones and Smith, though not necessarily in that order. The teller, who was an only child, earns the least. Smith, who married Brown's sister, earns more than the manager. What position does each man fill?

7. Twelve people came to celebrate their ten-year high school reunion. Each person shook hands once with all the other people. How many handshakes were exchanged?

8. There are three bags with red and blue marbles in them. One bag is labeled "All Red", one is labeled "All Blue", and the third is labeled "Red and Blue". However, each of the three bags is mislabeled. The problem is to figure out which bag really is which. You can't look into any of them, but you can reach into only one bag and remove one marble at random. Which bag would you reach into?

9. The **rectangular numbers** are whole numbers that are represented by certain rectangular array of dots. The first four rectangular numbers are shown:

   (a) How many dots will be in the sixth rectangular number?

   (b) How many dots will be in the n-th rectangular number?