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## Section 4.1: Mental Math

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- Mental Math uses the properties of whole numbers and compatible numbers. By compatible numbers, we mean those numbers that go together well with a particular operation.

- Commutative Properties:

$$a + b = b + a \quad \text{and} \quad a \cdot b = b \cdot a$$

- Associative Properties:

$$a + (b + c) = (a + b) + c \quad \text{and} \quad a(bc) = (ab)c$$

- Distributive Properties:

$$a(b + c) = ab + ac \quad \text{and} \quad a(b - c) = ab - ac$$

**Example 1:** Perform the following operations by using the compatible numbers and the properties of whole numbers.

(a)  $15 + (26 + 35) =$

(b)  $4 \times 9 \times 25 =$

(c)  $(8 \times 13) \times 25 =$

(d)  $23 \cdot 17 - 15 \cdot 23 =$

(e)  $97 + 58 =$

(f)  $168 \div 3 =$

- **Compensation:** Compensation is the process of reformulating a problem into one that is more easily obtained mentally.

- **Additive compensation:** increase one number by  $n$ , and decrease the second number by  $n$

**Example 2:**  $98 + 57 =$

- **Equal additions method:** In a subtraction problem, we add the same number to both numbers.

**Example 3:**  $93 - 48 =$

- **Left to Right Methods:** Researchers have found that those individuals that are excellent mental calculators utilize a left to right method.

**Example 4:**  $372 + 429 =$

- **Multiplying by special factors:** Since multiplying by 10 is easy, we consider the following:

$$5 = 10 \div 2 \qquad 25 = 100 \div 4 \qquad 99 = 100 - 1$$

**Example 5:**

(a)  $42 \times 5 =$

(b)  $36 \times 25 =$

(c)  $24 \times 99 =$

- **Range Estimation:** Often it is sufficient to know an interval or range that will contain an answer.

**Example 6:** Find a range for answers to these computations by using only the leading digits.

(a)  $583 + 356$

(b)  $342 \times 215$

- **Rounding:** The purpose of rounding is to replace complicated numbers with simpler numbers. There are several different rounding techniques and some may be more appropriate than others depending on the word problem situation. Some types of rounding are listed below.

- Round up
- Round down (also called **truncating**)
- Round a 5 up
- Round to the nearest even

**Example 7:** Estimate using the indicated method.

(a) Estimate  $2736 + 4832$  by rounding down to the nearest hundreds place.

(b) Estimate  $3750 - 1647$  by rounding to the nearest even hundreds place.

(c) Estimate  $839 - 428$  by round a 5 up to the nearest tens place.