Topic 1: Statements

- **<u>Statement</u>**: A statement is a declarative sentence that is true or false but not both. It cannot be a question, command, or opinion. It must be true or false (not ambiguous).

Examples:	It is raining	Not Examples:	What time is it?
	$(2)(3) = 6$		Ohio is the nicest state.
	$5 + 3 = 7$		The big dog
	Ohio is the largest state		This sentence is false.

- Notation: Statements are represented by lowercase letters such as p, q, r, s.

- **<u>Negation</u>**: The negation of a statement p, denoted $\sim p$, is a statement with the opposite truth value of p. (i.e.– if p is true then $\sim p$ is false, and if p is false then $\sim p$ is true.)

Example 1: If p is the statement "The sky is blue" then translate $\sim p$.

LOGICAL CONNECTIVES: Two or more statements can be connected to form **compound statements**. The four commonly used **logical connectives** are "and", "or", "if-then", and "if and only if".

- <u>AND</u>: The **conjunction** of p and q, denoted $p \wedge q$, is the statement "p and q".

- OR: The **disjunction** of p and q, denoted $p \vee q$, is the statement "p or q".

- IF–THEN: An **implication** (or **conditional statement**), denoted $p \to q$, is the statement "If p then q". (p is called the **hypothesis** and q is called the **conclusion**.)

- IF AND ONLY IF: The **biconditional statement**, denoted $p \leftrightarrow q$, is the statement "p if and only if q". ($p \leftrightarrow q$ is the conjunction of $p \to q$ and $q \to p$.)

Example 2: If p is false and q is true, find the truth values for each of the following.

(a) $\sim p \vee q$

(b) $p \wedge \sim q$

(c) $\sim (p \vee q)$

(d) $\sim (\sim p \wedge q)$

(e) $\sim q \wedge \sim p$

(f) $\sim p \rightarrow q$

(g) $\sim (p \rightarrow q)$

(h) $(p \vee q) \rightarrow (p \wedge q)$

(i) $(p \vee \sim p) \rightarrow p$

(j) $(p \vee q) \longleftrightarrow (p \wedge q)$

Example 3: Let r, s and t represent the following statements:

r	is "roses are red"
s	is "the sky is blue"
t	is "turtles are green"

Translate the following statements into English.

(a) $r \wedge s$

(b) $r \wedge (s \vee t)$

(c) $s \longrightarrow (r \wedge t)$

(d) $(s \wedge \sim t) \longrightarrow \sim r$

Example 4: Write the following arguments in symbolic form using $p, q, r, \sim, \wedge, \vee, \longrightarrow, \longleftrightarrow$, where p, q, and r are the given statements.

p	is "birds can fly"
q	is "horses can run"
r	is "fish can swim"

(a) Horses can run if and only if either birds cannot fly or fish can swim.

(b) If birds can fly and horses cannot run, then fish cannot swim.

(c) If it is not the case that either fish can swim or horses can run, then birds can fly.

- The **converse** of $p \to q$ is $q \to p$

- The **inverse** of $p \to q$ is $\sim p \to \sim q$

- The **contrapositive** of $p \to q$ is $\sim q \to \sim p$

Example 5: Consider the following implication:

If the bank was robbed, then I will not have any money.

(a) Find the inverse of the statement.

(b) Find the converse of the statement.

(c) Find the contrapositive of the statement.

- **logically equivalent**: Two statements are logically equivalent when they have the same truth tables.

Example 6: Using a truth table, determine which of the statements $p \longrightarrow q$, $q \longrightarrow p$, $\sim p \longrightarrow \sim q$, and $\sim q \longrightarrow \sim p$ are logically equivalent.

Example 7: Using a truth table, show that $\sim (p \longrightarrow q)$ and $p \wedge \sim q$ are logically equivalent.

Example 8: Using a truth table, show that $\sim (p \vee q)$ and $\sim p \wedge \sim q$ are logically equivalent.

Example 9: Using a truth table, show that $\sim (p \wedge q)$ and $\sim p \vee \sim q$ are logically equivalent.

- **DeMorgan's Laws for Statements:**

$$\sim (p \vee q) \;\Leftrightarrow\; \sim p \wedge \sim q$$

$$\sim (p \wedge q) \;\Leftrightarrow\; \sim p \vee \sim q$$

Example 10: Write the negation of each of the following statements.

(a) I do my homework and I pass my math class.

(b) I do my homework or I do not pass my math class.

(c) If I do my homework, then I will pass my math class.