Angle: An angle is the union of two line segments (or two rays) with a common endpoint, called a vertex.

Adjacent angles: Adjacent angles are two angles that share a vertex, have a common side, but whose interiors do not intersect.

Classification of Angles according to their measurements:

- **Acute angle**: angle measuring less than 90°
- **Right angle**: angle measuring 90°
- **Obtuse angle**: angle measuring more than 90° but less than 180°
- **Straight angle**: angle measuring 180°
- **Reflex angle**: angle measuring more than 180°
• **Vertical Angles:** Opposite angles formed by two intersecting lines are called **vertical angles.**

    IMPORTANT: Vertical angles always have the same measurement.

In the above figure, \( \angle 1 \) and \( \angle 3 \) are vertical angles; \( \angle 2 \) and \( \angle 4 \) are vertical angles.

• **Complementary Angles:** Two angles whose sum is 90° are called **complementary angles.** If \( A \) and \( B \) are complementary angles, then \( A \) is the **complement** of \( B \) and \( B \) is the complement of \( A \).

In the above figure, \( \angle 1 \) and \( \angle 2 \) are complementary angles.

• **Supplementary Angles:** Two angles whose sum is 180° are called **supplementary angles.** If \( A \) and \( B \) are supplementary angles, then \( A \) is the **supplement** of \( B \) and \( B \) is the supplement of \( A \).

In the above figure, \( \angle 1 \) and \( \angle 2 \) are supplementary angles.
**Example 1:** Find the measure of each marked angle.

(a)

(b)

(c)
ANGLES ASSOCIATED WITH PARALLEL LINES:

- **Corresponding Angles** have the same location relative to lines \( \ell, m \) and transversal \( t \). (IMPORTANT: \( \ell \parallel m \) if and only if corresponding angles formed by \( \ell, m, \) and \( t \) are congruent.)

  In Figure A-1, \( \angle 1 \) and \( \angle 5 \) are corresponding angles. The following pairs are also corresponding angles: \( \angle 2 \) and \( \angle 6 \); \( \angle 3 \) and \( \angle 7 \); \( \angle 4 \) and \( \angle 8 \).

- **Alternate Interior Angles** are nonadjacent angles formed by lines \( \ell, m, \) and transversal \( t \), the union of whose interiors contain the region between \( \ell \) and \( m \). (IMPORTANT: \( \ell \parallel m \) if and only if alternate interior angles formed by \( \ell, m, \) and \( t \) are congruent.)

  In Figure A-1, \( \angle 3 \) and \( \angle 6 \) are alternate interior angles. Likewise, \( \angle 4 \) and \( \angle 5 \) are also alternate interior angles.

- **Alternate Exterior Angles** are angles on the outer sides of two lines cut by a transversal, but on opposite sides of the transversal (IMPORTANT: \( \ell \parallel m \) if and only if alternate exterior angles formed by \( \ell, m, \) and \( t \) are congruent.)

  In Figure A-1, \( \angle 2 \) and \( \angle 7 \) are alternate exterior angles. Similarly, \( \angle 1 \) and \( \angle 8 \) are alternate exterior angles.

- **Interior Angles on the same side of the transversal** are interior angles whose interiors are the same. (IMPORTANT: \( \ell \parallel m \) if and only if the interior angles on the same side of the transversal are supplementary.)

  In Figure A-1, \( \angle 3 \) and \( \angle 5 \), as well as \( \angle 4 \) and \( \angle 6 \), are interior angles on the same side of the transversal.
Example 2: In the diagram below, $l \parallel m$ and $r \parallel s$. Find the measurement of each numbered angle.

NOTES:

- The sum of the angles inside a triangle is $180^\circ$.
- The sum of the angles inside a quadrilateral is $360^\circ$.

Example 3: In the diagram below, $BG \parallel EF$. Find the measure of each angle.
Exercises

In #1–#6 find the measure of each marked angle.

1. 

\[
\begin{align*}
(11x - 37)^\circ & \quad (7x+27)^\circ \\
\end{align*}
\]

4. 

\[
\begin{align*}
(5x + 3)^\circ & \quad (4x + 6)^\circ \\
\end{align*}
\]

2. 

\[
\begin{align*}
(x+1)^\circ & \quad (4x - 56)^\circ \\
\end{align*}
\]

5. 

\[
\begin{align*}
(8x + 17)^\circ & \quad (11x - 4)^\circ \\
\end{align*}
\]

3. 

\[
\begin{align*}
(5x + 11)^\circ & \quad (3x - 15)^\circ \\
\end{align*}
\]

6. 

\[
\begin{align*}
(3x-1)^\circ & \quad (4x+7)^\circ \\
\end{align*}
\]
7. Using the diagram below, name the relationship between the following pairs of angles, given \( m \parallel l \) and \( n \parallel p \).

![Diagram showing angles and lines]

a) \( \angle 1 \) and \( \angle 15 \)  
b) \( \angle 1 \) and \( \angle 5 \)  
c) \( \angle 1 \) and \( \angle 9 \)  
d) \( \angle 11 \) and \( \angle 10 \)  
e) \( \angle 12 \) and \( \angle 6 \)  
f) \( \angle 6 \) and \( \angle 8 \)  
g) \( \angle 3 \) and \( \angle 7 \)  
h) \( \angle 14 \) and \( \angle 10 \)  
i) \( \angle 7 \) and \( \angle 10 \)

8. In the diagram below, \( m \parallel n \). Find the measure of each numbered angle.

![Diagram showing angles and lines]

9. In the diagram below, \( t \parallel s \) and \( m \parallel n \). Find the measurement of each numbered angle.

![Diagram showing angles and lines]
10. In the diagram below, \( l \parallel m \) and \( r \parallel s \). Find the measurement of each numbered angle.

11. In the following figure, \( m \parallel n \) and \( r \perp s \). Given the angle measures indicated on the figure, find the measure of each lettered angle.

12. Find the measure of \( x \).

13. In the following figure, \( \overline{AB} \parallel \overline{CD} \). Find the measure of \( x \) and \( y \).
14. In the figure below, \( m \parallel n \). Find the measure of each labeled angle.

![Diagram](image)

15. Find the measure of \( a \) and \( b \).

![Diagram](image)

16. Find the of each labeled angle.

![Diagram](image)

17. In the figure below, \( m(\angle BFC) = 55^\circ \), \( m(\angle AFD) = 150^\circ \), and \( m(\angle BFE) = 120^\circ \). Determine the measures of \( \angle AFB \) and \( \angle CFD \).

![Diagram](image)
ANSWERS

1. Both angles are 139°
2. 48°, 132°
3. 54°, 126°
4. 48°, 42°
5. Both angles are 73°
6. 35°, 55°

7. (a) alternate exterior angles
   (b) alternate exterior angles
   (c) corresponding angles
   (d) interior angles on the same side of the transversal (supplementary angles)
   (e) alternate interior angles
   (f) corresponding angles
   (g) alternate interior angles
   (h) alternate interior angles
   (i) interior angles on the same side of the transversal (supplementary angles)

8. \( m(\angle 1) = 82°, m(\angle 2) = 58°, m(\angle 3) = 58°, m(\angle 4) = 82°, m(\angle 5) = 40°, \)
   \( m(\angle 6) = 58°, m(\angle 7) = 40°, m(\angle 8) = 140° \)

9. \( m(\angle 1) = 65°, m(\angle 2) = 115°, m(\angle 3) = 65°, m(\angle 4) = 115°, m(\angle 5) = 115°, \)
   \( m(\angle 6) = 65° \)

10. \( m(\angle 1) = 105°, m(\angle 2) = 75°, m(\angle 3) = 105°, m(\angle 4) = 105°, m(\angle 5) = 39°, \)
   \( m(\angle 6) = 36°, m(\angle 7) = 105°, m(\angle 8) = 39°, m(\angle 9) = 36°, m(\angle 10) = 105° \)

11. \( m(\angle a) = 40°, m(\angle b) = 75°, m(\angle c) = 55°, m(\angle d) = 145°, m(\angle e) = 35°, \)
    \( m(\angle f) = 125° \)

12. \( m(\angle x) = 83° \)

13. \( m(\angle x) = 50°, m(\angle y) = 60° \)

14. \( m(\angle a) = 8°, m(\angle b) = 64°, m(\angle c) = 108°, m(\angle d) = 72°, m(\angle e) = 55°, \)
    \( m(\angle f) = 53° \)

15. \( m(\angle a) = 58°, m(\angle b) = 104° \)

16. \( m(\angle a) = 70°, m(\angle b) = 130°, m(\angle c) = 100°, m(\angle d) = 120°, m(\angle e) = 20°, \)
    \( m(\angle f) = 20°, m(\angle g) = 80°, m(\angle h) = 60°, m(\angle i) = 70° \)

17. \( m(\angle AFB) = 60°, m(\angle CFD) = 35° \)