Routing problem: A routing problem is concerned with finding ways to route the delivery of good and/or services to an assortment of destinations. Once we determine there is a route, we will be concerned with finding the best route.

Graph: A graph is a collection of points, called vertices, and lines, called edges. An edge does not need to be a straight line, but must connect two vertices. The following is an example of a graph:

- This graph has 5 vertices called A, B, C, D, and E. The vertex set of the previous example is
  \[ V = \{A, B, C, D, E\} \].
- Each edge can be described by listing (in any order) the pair of vertices that are connected by an edge. The edges of this graph are AB, BC, CD, ED, AE, AD, EC, EE, and CD. The edge set of the previous example is
  \[ E = \{AB, BC, CD, ED, AE, AD, EC, EE, CD\} \].
  * Note that it is possible for two edges to connect the same pair of vertices, as is the case with CD, which is a double-edge. In general, we refer to such edges as multiple edges.
- Loop: If an edge connects a vertex back to itself, which is also allowable, then it is called a loop. In the above example EE is a loop.
- Sometimes edges cross each other that are not vertices of the graph. In the above example, the point where AD and EC cross is NOT a vertex of the graph.
- Edges do not have a direction. Therefore, AB could also be written as BA.
- Given a set of vertices and an edge set one can set up the graph.
Example 1: Consider the graph with $V = \{X, Y, U, W\}$ and $E = \{XY, XW, WU, XY, UU, WY\}$. Draw two different pictures of the graph.

For the next definitions, consider the following graph:

- **isolated vertex**: An isolated vertex is a vertex with no edges connecting it to another vertex.

- **adjacent vertices**: Two vertices are adjacent if there is an edge joining them. In the above graph, vertices $E$ and $B$ are adjacent; vertices $C$ and $E$ are not adjacent. Because of the loop $EE$, we say that $E$ is adjacent to itself.

- **adjacent edges**: Two edges are adjacent if they share a common vertex. In the above graph, edges $AB$ and $AD$ are adjacent; edges $AB$ and $DE$ are not adjacent.

- **degree of the vertex**: The degree of a vertex is the number of edges meeting at that vertex. The degree of vertex $A$ is denoted $\text{deg}(A)$. A loop counts twice toward the degree. In the above graph, $\text{deg}(A) = 3$, and $\text{deg}(E) = 4$.

- **even and odd degrees vertices**: Even vertices are vertices with an even degree. Odd vertices are vertices with an odd degree. In the above example, the graph has two even vertices ($D$ and $E$) and six odd vertices ($A, B, C, F, G$, and $H$).

- **walk**: A walk in a graph is a sequence of vertices, each linked to the next vertex by a specific edge of the graph. In the above graph, $A \rightarrow D \rightarrow E \rightarrow D$, also denoted $A, D, E, D$, is an example of a walk.
• **path:** A path is an open trip. A path is a sequence of vertices with the property that each vertex in the sequence is adjacent to the next one. While a vertex can appear on the path more than once, an edge can be a part of a path only once.

• **circuit:** A circuit is a closed trip. A circuit is a path that starts and ends at the same vertex.

![Graph Diagram]

* The **length** of a path is the number of edges in the path.

* $A, B, E, D$ is a path of length 3 from $A$ to $D$ consisting of the edges $AB$, $BE$, and $ED$.

* $A, B, C, A, D, E$ is a path of length 5 from $A$ to $E$. This path visits vertex $A$ twice, but no edge is repeated.

* $A, C, D, E$ is NOT a path, since there is no edge connecting $C$ to $D$.

* $A, B, C, B, F$ is a path of length 4 from $A$ to $F$.

* $A, B, C, A$ is an example of a circuit of length 3.

* The $EE$ loop is considered a circuit of length 1.

• **connected graph:** A graph is connected if any two of its vertices can be joined by a path. In other words, the graph is all in one piece. The previous graph was an example of a connected graph.

• **disconnected graph:** A graph is disconnected if it is not connected. A graph that is disconnected is made up of pieces that are connected. These pieces are called the components.

**Example 2:** Determine if the following graph is connected or disconnected. If disconnect, determine the number of components.

![Graph Diagram]
Example 3: Consider the following graph.

(a) List all edges adjacent to $AF$.

(b) List all vertices adjacent to $C$.

(c) Determine the degree of $A$.

(d) Find a path of length 4 that begins at $A$ and ends at $E$.

(d) Find a circuit of length 4 that begins at $B$.

(e) Give an example of a walk of length 4 that is NOT a path.
• **Bridge:** In a connected graph, a bridge is an edge that if it is removed, the graph becomes disconnected. A bridge is also called a cut edge.

**Example 4:** Identify a bridge, or cut edge, in the following graph.

![Graph Example](image)

**Example 5:** A map of downtown Royalton is given below, showing the Royalton River running through the downtown area and the three islands (A, B, and C) connected to each other and both banks by eight bridges. The local athletic club wants to design the route for a marathon through the downtown area. Draw a graph that models the layout of Royalton.

![Map of Royalton](image)

**Example 6:** A mail carrier must deliver mail on foot along the streets of the Green Hills subdivision shown below. The mail carrier must make two passes on every block that has houses on both sides of the street (once for each side of the street), but only one pass on blocks that have houses on only one side of the street. Draw a graph that models this situation.

![Map of Green Hills](image)
• **Euler path:** An **Euler path** in a connected graph is a path that travels through ALL the edges of the graph exactly once.

• **Euler circuit:** An **Euler circuit** in a connected graph is a circuit that travels through every edge exactly once.
  - A connected graph cannot have both an Euler path and an Euler circuit. It can have one or the other or neither.