15.4: Impossibilities of Apportionment

- **Quota Method:** A quota method of apportionment uses the standard divisor d, and apportions a number of seats to each state either equal to the standard quota Q rounded up to the nearest integer, or the value of Q rounded down to the nearest integer.

  * Hamilton's Method is a quota method.

- **Divisor Method:** A divisor method of apportionment uses a modified divisor and not the standard divisor.

  * Jefferson's Method and Weber's Method are divisor methods.

- **The Quota Rule:** The integer number of things apportioned to each state must be either the standard quote Q rounded down to the nearest integer, or the standard quota Q rounded up to the nearest integer.

  * Hamilton's Method satisfies the quota rule. In fact, any quota method satisfies the quota rule.
  * Jefferson' Method and Weber's Method do not satisfy the quota rule.

- **The Alabama Paradox:** The situation in which an increase in the number of objects being apportioned actually forces a state to lose one of those objects is known as the Alabama Paradox.

  * After 1880 census, government workers calculated the different Hamilton apportionments for the House of Representatives sizes from 275 to 350 seats. It was found that if the House had 299 seats then Alabama would receive 8 seats. But, if the House had 300 seats then Alabama would receive 7 seats.

Example 1: A longtime resident of the Highwood School District has given one additional computer to the district. The gift brings the total number of computers to be apportioned to 110 computers. Use the Hamilton Method to apportion the 110 computers.

School	Enrollment	Standard Quota Q	Rounded-down Q	Computer Apportioned
Applegate	335			
Bayshore	456			
Claypool	298			
Delmar	567			
Edgewater	607			
Totals	2263			

- **The Population Paradox:** The population paradox occurs when, based on updated population figures, a reapportionment of a fixed number of seats causes a state to lose a seat to another state, although the percent increase in the population of the state that loses the seat is bigger than the percent increase in the population of the state that gains the seat.

Example 2: Use the Hamilton Method to apportion 11 legislative seats to three states with the given populations. Show that the population paradox occurs when the Hamilton method is used to apportion the 11 legislative seats again, after the population figures are revised by a census. The populations are given in thousands.

State	a	b	c
Initial population	55	125	190
Revised population	62	150	218

- **The New States Paradox:** The new states paradox occurs when a reapportionment of an increased number of seats, necessary due to the addition of a new state, causes a shift in the apportionment of the original states.

  * Occurred in 1907 when Oklahoma joined the Union.

Example 3: Use the Hamilton Method to apportion 16 legislative seats to two states with the given populations. Show that the new states paradox occurs if a new state with the indicated population and the appropriate number of additional seats in included in a second Hamilton method apportionment. The appropriate number of seats to add for the second apportionment is the state's standard quota of the new state, computed using the original two state standard divisor, rounded down to the nearest integer. The populations are given in thousands.

State	Original State a	Original State b	New State c
Population	134	52	39

Summary of the four fairness standards

	Quota Rule	Alabama Paradox	Population Paradox	New States Paradox
Hamilton Method	satisfied	allows	allows	allows
Jefferson Method	can violate	does not occur	does not occur	does not occur
Webster Method	can violate	does not occur	does not occur	does not occur

- **The Balinski and Young Impossibility Theorem (1980):** Any apportionment method that always satisfies the quota rule will permit the possibility of a paradoxical apportionment. Likewise, any apportionment method that does not permit the possibility of a paradoxical apportionment will fail to always satisfy the quota rule.