Section 12.2: Meas of Central Tendency

- **Mode**: score or number which occurs most often.
  - There can be more than one mode; however, if each number appears equally often, then there is no mode.
  - Mode is not affected by other scores.

- **Median**: middle score or “halfway” point.
  To find median: Suppose $x_1, x_2, \cdots, x_n$ are the scores listed in increasing order.
  - If $n$ is odd, the median is the middle score in the list; namely, the $\frac{n+1}{2}$ score.
  - If $n$ is even, the median is the average of the two middle scores; namely, average the $\frac{n}{2}$ and $\frac{n}{2} + 1$ scores.

**Example 1**: Find the median of 37, 24, 35, 23, 82, 45, 56, 32, 57,

**Example 2**: Find the median of 42, 23, 36, 56, 39, 45, 25, 57
• **Mean**: arithmetic average of the scores, denoted by \( \bar{x} \).

\[
\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \text{add all scores together}
\text{total number of scores}
\]

– Mean is affected by extremely high or extremely low scores.

**Example 3**: Find the mean of each set of numbers.

(a) 83, 82, 81, 79

(b) 83, 82, 81, 21

• **Weighted Mean**: The weighted mean of \( n \) numbers \( x_1, x_2, \ldots, x_n \) that are weighted by the respective factors \( f_1, f_2, \ldots, f_n \) is given by the formula

\[
\bar{w} = \frac{x_1 f_1 + x_2 f_2 + \cdots + x_n f_n}{f_1 + f_2 + \cdots + f_n}
\]

**Example 4**: Rory received the following grade report. Calculate her semester grade point average.

<table>
<thead>
<tr>
<th>Course</th>
<th>Grade</th>
<th>Credit hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>MATH 12002</td>
<td>B</td>
<td>5</td>
</tr>
<tr>
<td>ENG 11011</td>
<td>C</td>
<td>3</td>
</tr>
<tr>
<td>CHEM 10050</td>
<td>A</td>
<td>3</td>
</tr>
<tr>
<td>FR 13202</td>
<td>B</td>
<td>4</td>
</tr>
</tbody>
</table>
• **Box Plot**: A graphical representation of the median and other scores.

To construct a box plot, we need to find 5 items

1. lowest score
2. median
3. highest score
4. **lower quartile**: median of all items below median.
5. **upper quartile**: median of all items above median.

Next, we plot these on a number line.

- Make a box from the lower to upper quartile.
- Use a vertical line inside this box to represent the median.
- Connect the lowest score to lower quartile with a horizontal line segment. Do the same with upper quartile and highest score.

**Example 5**: Create a box plot for the following set of data points:

4.3, 4.9, 4.9, 5.1, 5.2, 5.2, 5.3, 5.3, 5.3, 5.4, 5.4, 5.6, 5.6, 5.7, 5.8, 5.8, 5.9, 6.1, 6.2, 6.9
• Modified Box Plot: Differs from a box plot by identifying outliers.

Interquartile Range (IQR): difference between the upper and lower quartile.

Outlier: any data point which lies more than 1.5 IQR units below lower quartile or more than 1.5 IQR units above upper quartile. Outliers are denoted by an asterisk.

Example 6: Identify any outliers for the data set in Example 5 and create a modified box plot for this data.
EXERCISES

1. *Time* published an article on the academic ability of college athletes. The article noted that some of the most successful athletic programs have athletes with very good college board scores. Assume that the following sample data are typical of college board scores for Kent State University football players:

   1100, 970, 1000, 1250, 880, 1300, 1050, 900, 950, 1120

   (a) Compute the mean, median and mode
   (b) Compute the lower quartile and the upper quartile
   (c) Compute the range and interquartile range
   (d) State whether or not there are any outliers in this data set.

2. Consider the following set of quiz scores:

   1, 2, 2, 3, 4, 7, 7, 7, 8, 9

   Compute the following:

   (a) mean  (b) median  (c) mode  (d) lower quartile  (e) upper quartile  (f) range  (g) Construct a box-plot for this data.

3. Listed below are the ages at inauguration of 18 U.S. presidents.

   69, 64, 62, 61, 60, 56, 56, 55, 55, 54, 54, 52, 51, 51, 51, 46, 43

   (a) Compute the median
   (b) Compute the upper quartile
   (c) Compute the lower quartile
   (d) Compute the interquartile range (IQR)
   (e) Determine if there are any outliers. Is so, what are they? Show all work.
   (f) Construct a modified box plot for this data.
4. The final medal standing for the top 15 countries at the 1996 Summer Olympics are listed below.

<table>
<thead>
<tr>
<th>Country</th>
<th>Medals</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>101</td>
</tr>
<tr>
<td>Germany</td>
<td>65</td>
</tr>
<tr>
<td>China</td>
<td>63</td>
</tr>
<tr>
<td>Australia</td>
<td>50</td>
</tr>
<tr>
<td>France</td>
<td>41</td>
</tr>
<tr>
<td>Italy</td>
<td>37</td>
</tr>
<tr>
<td>South Korea</td>
<td>35</td>
</tr>
<tr>
<td>Cuba</td>
<td>27</td>
</tr>
<tr>
<td>Ukraine</td>
<td>25</td>
</tr>
<tr>
<td>Canada</td>
<td>23</td>
</tr>
<tr>
<td>Hungary</td>
<td>22</td>
</tr>
<tr>
<td>Romania</td>
<td>21</td>
</tr>
<tr>
<td>Netherlands</td>
<td>20</td>
</tr>
<tr>
<td>Poland</td>
<td>19</td>
</tr>
<tr>
<td>Spain</td>
<td>17</td>
</tr>
</tbody>
</table>

Compute the following:

(a) median
(b) lower quartile
(c) upper quartile
(d) range
(e) interquartile range (IQR)
(f) Are there any outliers? If so, what are they? Show all work.
(g) Construct a Modified Box-plot for this data.
ANSWERS

1. (a) mean = 1052, median = 1025, no mode
   (b) lower quartile = 950, upper quartile = 1120
   (c) range = 420, IQR = 170
   (d) no outliers

2. (a) mean = 5
   (b) median = 5.5
   (c) mode = 7
   (d) lower quartile = 2
   (e) upper quartile = 7
   (f) range = 8
   (g)

   ![Boxplot](image)

3. (a) 54.5
   (b) 60
   (c) 51
   (d) 9
   (e) no outliers
   (f)

   ![Boxplot](image)

4. (a) 27
   (b) 21
   (c) 50
   (d) 84
   (e) 29
   (f) 101 is an outlier
   (g)

   ![Boxplot](image)