
MATH 11008: Odds and Expected Value

- **Odds:** Odds compares the number of favorable outcomes to the number of unfavorable outcomes.

Suppose all outcomes in a sample space are equally likely where a of them are favorable to the event E and the remaining b outcomes are unfavorable to the event E .

* **Odds in favor of E :** are a to b , denoted $a : b$. In other words, $n(E) : n(\overline{E})$

* **Odds against:** are b to a , denoted $b : a$. In other words, $n(\overline{E}) : n(E)$

- Odds ratio are normally simplified. For example, it is preferable to express odds as $12 : 1$ rather than $48 : 4$.

Example 1: A local baseball team has won 13 games and lost 2 games.

- What is the baseball team's odds in favor of winning the next game?
- What is the baseball team's odds against winning the next game?

Example 2: A card is drawn at random from a standard deck. Find

- Odds in favor of drawing a face card.
- Odds against drawing a diamond.
- Odds in favor of drawing the ace of spades.
- Odds against drawing a 2, 3 or 4.

Example 3: Suppose that the odds in favor of an event E are $3 : 7$. Find $P(E)$.

- **Expected Value:** Suppose that the outcomes of an experiment are real numbers called $v_1, v_2, \dots, v_n$ and suppose that the outcomes have probability $p_1, p_2, \dots, p_n$. The **expected value of the experiment** is the sum

$$v_1 \cdot p_1 + v_2 \cdot p_2 + \cdots + v_n \cdot p_n$$

Stated in other words,

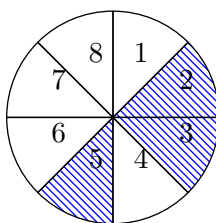
$$\text{expected value} = v_1 \cdot P(v_1) + v_2 \cdot P(v_2) + \cdots + v_n \cdot P(v_n).$$

Example 4: According to a publisher's records, 20% of the books published break even, 30% lose \$1000, 25% lose \$10,000, and 25% earn \$20,000. When a book is published, what is the expected income for the book?

Example 5: A player pays \$3 to play the following game: He tosses three fair coins and receives back “payoffs” of \$1 if he tosses no heads, \$2 for one head, \$3 for two heads, and \$4 for three heads. Find the player's expected net winnings for this game.

Exercises

1. Donna has purchased six tickets for an office raffle where the winner will receive a cellular phone. If 51 tickets were sold altogether and each has an equal chance of winning, what are the odds against Donna's winning the phone?
2. Suppose the probability of rain tomorrow is 0.13. Give the odds against rain tomorrow.
3. If $P(E) = 0.37$, find
 - (a) the odds in favor of E
 - (b) the odds against E
4. If the odds in favor of event E are 12 : 9, find $P(E)$.
5. If the odds against event E are 10 : 3, find $P(E)$.
6. A preschool group contains eight girls and five boys. A child is randomly selected to be first in line and let E be the event that the one selected is a girl.
 - (a) Find the odds in favor of event E
 - (b) Find $P(E)$
7. A card is drawn at random from a standard deck. Find
 - (a) Odds in favor of drawing a red card.
 - (b) Odds in favor of drawing a spade.
 - (c) Odds against drawing a 7.
 - (d) Odds against drawing an Ace, King, or Queen.
8. Consider the given spinner.



- (a) Find the odd in favor of it landing on an even number.
 - (b) Find the odd against it landing on a shaded number.
 - (c) Find the odds against it landing on a shaded even number.
 - (d) Find the odds in favor of it landing on a shaded number or an even number.
9. Five fair coins are tossed. find the expected number of heads.

10. From the following data, compute the expected value.

Payoff	-2	0	2	3
Probability	0.3	0.1	0.4	0.2

11. In a certain state lottery, a player chooses a three digit number, in a specific order, where the leading digit may be a zero. The lottery operator randomly selects a three digit sequence, and any player matching this sequence receives a payoff of \$600. What is a fair cost to play this game?
12. One simple type of roulette is played with an ivory ball and a wheel set in motion. The wheel contains thirty-eight compartments. Eighteen of the compartments are red, eighteen are black, one is labeled zero, and one is labeled double zero. The zero and double zero are neither black nor red. In this case, the player places \$1 on either red or black. If the player picks the correct color of the compartment in which the ball finally lands, the payoff is \$2; otherwise the payoff is zero. Find the expected winnings.
13. A certain game consists of rolling a single fair die, and pays off as follows: \$3 for rolling a 6; \$2 for rolling a 5; \$1 for rolling a 4; and no payoff otherwise.
- (a) Find the expected winnings for this game.
- (b) what is a fair price to pay to play this game.
14. A certain game consists of rolling a single fair die, and pays off as follows: If an even number of spots turns up, you receive that many dollars. But if an odd number of spots turns up, you must pay that many dollars.
- (a) Find the expected net winnings of this game?
- (b) Is this game fair, or unfair against the player, or unfair in favor of the player?
15. A certain game involves tossing 3 fair coins, and it pays 10 cents for 3 heads, 5 cents for 2 heads, and 3 cents for 1 head. Is 5 cents are fair price to pay to play this game?
16. In a MATH 11008 class, the probabilities have been determined for various numbers of absences on any given day. These values are shown in the table below. Find the expected number of absences on a given day.

Number absent	0	1	2	3	4
Probability	.12	.32	.35	.14	.07

17. A college foundation raises funds by selling raffle tickets for a new car worth \$36,000.
- (a) If 600 tickets are sold for \$120 each, determine the expected net winnings of a person buying one of the tickets.
- (b) If 720 tickets are sold for \$120 each, determine the expected net winnings of a person buying one of the tickets.

Answers

1. $15 : 2$
2. $87 : 13$
3. (a) $37 : 63$
(b) $63 : 37$
4. $\frac{12}{21}$
5. $\frac{3}{13}$
6. (a) $8 : 5$
(b) $\frac{8}{13}$
7. (a) $1 : 1$
(b) $1 : 3$
(c) $12 : 1$
(d) $10 : 3$
8. (a) $1 : 1$
(b) $5 : 3$
(c) $7 : 1$
(d) $3 : 1$
9. $\frac{5}{2}$
10. 0.80
11. \$0.60
12. expected loss of \$0.053 per play.
13. (a) \$1
(b) \$1
14. (a) \$0.50
(b) unfair in favor of the player.
15. no (expected losses of $\frac{3}{4}$ cents)
16. 1.72
17. (a) $-\$60$
(b) $-\$70$