Sections 2.1/2.2: Sets

- A set is a collection of objects.
- An element is an object in the set.
- For a set to be useful, it must be well defined. This means that if a set and an element are given, it must be possible to determine whether or not the element belongs to the set.

Two types of set notation:

1. Listing method: Listing the elements separated by commas. \(\{\text{Oklahoma, Ohio, Oregon}\}\).
2. Set-Builder Notation: \(\{x \mid x \text{ is a US state which begins with the letter O}\}\).

Notation:

- \(\in\) denotes that an object is in the set.
- \(\notin\) denotes that the object is NOT in the set.
- \(\emptyset\) or \(\{\}\) denote the empty set. The empty set is a set with no elements.

Example 1: Fill in each blank with either \(\in\) or \(\notin\) to make the following statements true.

(a) \(4 \_ \{2, 4, 6, 8, 10\}\)
(b) \(9 \_ \{x \mid x \text{ is a multiple of 4}\}\)

Other definitions:

- Cardinality: The number of elements in a set \(A\) is called the cardinal number or cardinality of set \(A\), denoted \(n(A)\).
- Equal sets: Two sets \(A\) and \(B\) are equal, denoted \(A = B\), if and only if they have the same elements.
- Equivalent sets: Two sets \(A\) and \(B\) are equivalent, denoted \(A \sim B\), if and only if they have the same number of elements.
• **Subset of a set:** Set $A$ is said to be a subset of $B$, denoted $A \subseteq B$, if and only if every element of $A$ is also an element of $B$.

• **Proper subset:** $A$ is a proper subset of $B$, denoted $A \subset B$, if $A \subseteq B$ and $B$ has an element that is not in $A$.

• **Disjoint sets:** $A$ and $B$ are disjoint if they have no elements in common.

**Example 2:** Write true or false for each of the following statements. If false, tell why and explain how you can correct the statement to make it true.

(a) $6 \notin \{3, 7, 8, 9\}$

(b) $\{4\} \in \{1, 2, 3, 4\}$

(c) $\{1, 2, 3, 4\} = \{x | x \text{ is a counting number less than 5}\}$

(d) $\{7\} \subset \{1, 7\}$

(e) $\{7\} \subseteq \{1, 7\}$

(f) $\{3, 5, 7\} \subset \{3, 5, 7\}$

• **Number of subsets:** If a set has $n$ elements, then it has $2^n$ subsets.

• **Number of proper subsets:** If a set has $n$ elements, then it has $2^n - 1$ proper subsets.

**Example 3:** Determine the number of subsets for $\{2, 3, 4\}$. List all the proper subsets.

**Example 4:** Determine the number of proper subsets for $\{a, e, i, o, u\}$