MATH 11009: Integer Exponents

The Laws of Exponents

For $x \neq 0$, $y \neq 0$

- $x^m x^n = x^{m+n}$
- $(x^m)^n = x^{mn}$
- $\dfrac{x^m}{x^n} = x^{m-n}$
- $(xy)^n = x^n y^n$
- $\left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$
- $x^0 = 1$
- $x^{-n} = \dfrac{1}{x^n}$
- $\dfrac{1}{x^{-n}} = x^n$
- $\left(\dfrac{x}{y}\right)^{-n} = \left(\dfrac{y}{x}\right)^n$

Example 1. Simplify: $(3x^4y^3)^2(7xy^5)$

Solution.

$$(3x^4y^3)^2(7xy^5) = (9x^8y^6)(7xy^5)$$

$$= \boxed{63x^9y^{11}}$$

Example 2. Simplify: $(2x^{-3}y^4)^3(3x^2y^{-1})^{-2}$

Solution.

$$(2x^{-3}y^4)^3(3x^2y^{-1})^{-2} = (8x^{-9}y^{12})(3^{-2}x^{-4}y^2)$$

$$= 8x^{-13}y^{14} \cdot 3^{-2}$$

$$= \frac{8y^{14}}{3^2x^{13}}$$

$$= \boxed{\frac{8y^{14}}{9x^{13}}}$$

Example 3. Simplify: $\dfrac{18x^{-5}y^{-2}}{10x^{-2}y^{-8}}$

SOLUTION.

$$\frac{18x^{-5}y^{-2}}{10x^{-2}y^{-8}} = \frac{18x^2y^8}{10x^5y^2}$$

$$= \boxed{\frac{9y^6}{5x^3}}$$

Example 4. Simplify: $\dfrac{14x^{-3}y^{-2}}{24x^{-8}y^3}$

SOLUTION.

$$\frac{14x^{-3}y^{-2}}{24x^{-8}y^3} = \frac{14x^8}{24x^3y^3y^2}$$

$$= \boxed{\frac{7x^5}{12y^5}}$$

Example 5. Simplify: $\dfrac{(2x^{-3}y^4)^{-1}}{6x^{-5}y^{-2}}$

SOLUTION.

$$\frac{(2x^{-3}y^4)^{-1}}{6x^{-5}y^{-2}} = \frac{2^{-1}x^3y^{-4}}{6x^{-5}y^{-2}}$$

$$= \frac{x^3x^5y^2}{2^1 \cdot 6y^4}$$

$$= \boxed{\frac{x^8}{12y^2}}$$

Example 6. Find: $2^{-3} + 3^{-2} - 4^{-1} =$

SOLUTION.

$$2^{-3} + 3^{-2} - 4^{-1} = \frac{1}{2^3} + \frac{1}{3^2} - \frac{1}{4^1}$$

$$= \frac{1}{8} + \frac{1}{9} - \frac{1}{4}$$

$$= \frac{9}{72} + \frac{8}{72} - \frac{18}{72}$$

$$= \boxed{-\frac{1}{72}}$$

Example 7. Find: $(-2)^{-4} - (-3)^{-1} + (-2)^{-3} =$

SOLUTION.

$$(-2)^{-4} - (-3)^{-1} + (-2)^{-3} = \frac{1}{(-2)^4} - \frac{1}{(-3)^1} + \frac{1}{(-2)^3}$$

$$= \frac{1}{16} - \frac{1}{-3} + \frac{1}{-8}$$

$$= \frac{1}{16} + \frac{1}{3} - \frac{1}{8}$$

$$= \frac{3}{48} + \frac{16}{48} - \frac{6}{48}$$

$$= \boxed{\frac{13}{48}}$$

Example 8. Find: $\left(\frac{4}{3}\right)^{-1} + \left(\frac{2}{5}\right)^{-2} =$

SOLUTION.

$$\left(\frac{4}{3}\right)^{-1} + \left(\frac{2}{5}\right)^{-2} = \frac{4^{-1}}{3^{-1}} + \frac{2^{-2}}{5^{-2}}$$

$$= \frac{3^1}{4^1} + \frac{5^2}{2^2}$$

$$= \frac{3}{4} + \frac{25}{4}$$

$$= \frac{28}{4}$$

$$= \boxed{7}$$

Example 9. Find: $\left(\frac{2}{3}\right)^{2} - \left(\frac{3}{4}\right)^{-1} + \left(\frac{3}{4}\right)^{-2} =$

SOLUTION.

$$\left(\frac{2}{3}\right)^{2} - \left(\frac{3}{4}\right)^{-1} + \left(\frac{3}{4}\right)^{-2} = \frac{4}{9} - \frac{3^{-1}}{4^{-1}} + \frac{3^{-2}}{4^{-2}}$$

$$= \frac{4}{9} - \frac{4^1}{3^1} + \frac{4^2}{3^2}$$

$$= \frac{4}{9} - \frac{4}{3} + \frac{16}{9}$$

$$= \frac{4}{9} - \frac{12}{9} + \frac{16}{9}$$

$$= \boxed{\frac{8}{9}}$$

EXERCISES

Simplify:

1. $(2x^{-3}y^2)^3$
2. $(4x^{-1}y^{-3})^2(5x^{-1}y^3)^{-1}$
3. $(4x^3y^2)^2(2x^2y^3)^{-3}$
4. $(3x^{-2}y^4)^{-2}$
5. $(6x^4y^{-3})^{-1}(8x^6y^{-2})$
6. $\dfrac{6x^5y^{-2}}{15x^{-3}y^8}$
7. $\dfrac{(4x^{-3}y^5)^{-1}}{8x^5y^{-3}}$
8. $\dfrac{9x^5y^{-3}}{24x^7y^8}$

Find:

9. $3^{-1}+2^{-2}=$
10. $(-4)^{-2}-2^{-1}=$
11. $2^2-2+2^{-1}-2^{-2}=$
12. $(-3)^2+3-3^{-1}+3^{-2}=$
13. $2^{-1}+3^{-1}-4^{-1}=$
14. $(-2)^{-1}-(-2)^{-2}+(-2)^{-3}=$
15. $\left(\dfrac{2}{3}\right)^{-2}+\left(\dfrac{4}{5}\right)^{-1}=$
16. $\left(\dfrac{1}{3}\right)^{-1}+3^{-1}+\left(\dfrac{1}{3}\right)^{-2}+3^{-2}=$
17. $\left(\dfrac{2}{3}\right)^{-2}+\left(\dfrac{3}{2}\right)^{-2}-\left(\dfrac{1}{3}\right)^{2}-\left(\dfrac{1}{2}\right)^{2}=$
18. $\left(\dfrac{1}{2}\right)^{2}-\dfrac{1}{2}+\left(\dfrac{1}{2}\right)^{-1}-\left(\dfrac{1}{2}\right)^{-2}=$

ANSWERS

1. $\dfrac{8y^6}{x^9}$
2. $\dfrac{16}{5xy^9}$
3. $\dfrac{2}{y^5}$
4. $\dfrac{x^4}{9y^8}$
5. $\dfrac{4x^2y}{3}$
6. $\dfrac{2x^8}{5y^{10}}$
7. $\dfrac{1}{32x^2y^2}$
8. $\dfrac{3}{8x^2y^{11}}$
9. $\dfrac{7}{12}$
10. $-\dfrac{7}{16}$
11. $\dfrac{9}{4}$
12. $\dfrac{106}{9}$
13. $\dfrac{7}{12}$
14. $-\dfrac{7}{8}$
15. $\dfrac{7}{2}$
16. $\dfrac{112}{9}$
17. $\dfrac{7}{3}$
18. $-\dfrac{9}{4}$