MATH 11009:   Factoring

Example 1.   Completely factor: $x^2 + 2xy - 8y^2$

SOLUTION.

$$x^2 + 2xy - 8y^2 = \boxed{(x - 2y)(x + 4y)}$$

Example 2.   Completely factor: $x^2y^2 + 8xy + 15$

SOLUTION.

$$x^2y^2 + 8xy + 15 = \boxed{(xy + 3)(xy + 5)}$$

Example 3.   Completely factor: $3x^2 - 12x - 36$

SOLUTION.

$$3x^2 - 12x - 36 = 3(x^2 - 4x - 12) = \boxed{3(x + 2)(x - 6)}$$

THE SUM/DIFFERENCE OF TWO SQUARES

$$a^2 - b^2 = (a - b)(a + b)$$
$$a^2 + b^2 = \text{prime}$$

Example 4.   Completely factor: $4x^2 - 25$

SOLUTION.

$$4x^2 - 25 = \boxed{(2x - 5)(2x + 5)}$$

Example 5.   Completely factor: $9x^2 + 49$

SOLUTION.

$$9x^2 + 49 = \boxed{\text{prime}}$$

Example 6. Completely factor: $5 + 9x - 2x^2$

SOLUTION.

$$5 + 9x - 2x^2 = \boxed{(5 - x)(1 + 2x)}$$

$$\text{OR}$$

$$5 + 9x - 2x^2 = -2x^2 + 9x + 5 = -1(2x^2 - 9x - 5) = \boxed{-1(2x + 1)(x - 5)}$$

Example 7. Completely factor: $8x^3 + 4x^2 - 18x - 9$

SOLUTION. Use factoring by grouping:

$$8x^3 + 4x^2 - 18x - 9 = 4x^2(2x + 1) - 9(2x + 1)$$

$$= (2x + 1)(4x^2 - 9)$$

$$= \boxed{(2x + 1)(2x - 3)(2x + 3)}$$

THE SUM/DIFFERENCE OF TWO CUBES

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example 8. Factor: $x^3 - 64$

SOLUTION. Factor the difference of two cubes using the formula $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$:

$$x^3 - 64 = x^3 - 4^3$$

$$= (x - 4)(x^2 + 4x + 4^2)$$

$$= \boxed{(x - 4)(x^2 + 4x + 16)}$$

EXERCISES

Completely factor the following:

Section 1

1. $36x^2y + 8xy^3$

6. $2x^4y - 2x^4y^2 + 2x^3y^2$

2. $-40x^8y^7 - 16x^9y^5$

7. $8x^4y^2 - 16xy^3$

3. $15x^3y^2 - 25x^2y^3$

8. $4x^3y - 12x^2y^2 + 8x^2y$

4. $-3x^3y - 9x^2$

9. $9x - 27y - 81$

5. $-15x^5y^4 + 10x^3y^5$

10. $6x^3y + 15x^2y^2 + 3x^2y$

Section 2

1. $x^2 + 7x + 6$

2. $x^2 + 6x + 8$

3. $x^2 + 9x + 20$

4. $x^2 + 13x + 30$

5. $x^2 - 8x + 15$

6. $x^2 - 9x + 14$

7. $x^2 - 10x + 9$

8. $x^2 - 6x + 9$

9. $x^2 - 3x - 18$

10. $x^2 - x - 30$

11. $x^2 - 5x - 14$

12. $x^2 - 16x + 48$

13. $x^2 - 10x + 21$

14. $x^2 - 4x - 21$

15. $x^2 - 4x - 32$

16. $x^2 - 16x + 63$

17. $x^2 + 18x + 81$

18. $x^2 - 12x + 36$

19. $x^2 - 2x - 48$

20. $x^2 + x - 42$

Section 3

1. $x^2 + 8xy + 15y^2$

2. $x^2 + 6xy + 8y^2$

3. $x^2 - 2xy + y^2$

4. $x^2 - 11xy + 30y^2$

5. $x^2 + 7xy + 10y^2$

6. $x^2 + xy - 20y^2$

7. $x^2 - 7xy + 12y^2$

8. $x^2 - 5xy + 6y^2$

9. $x^2y^2 + 3xy - 4$

10. $x^2y^2 + xy - 2$

11. $x^2y^2 - 6xy + 9$

12. $x^2y^2 - xy - 6$

13. $x^2y^2 - 4xy - 21$

14. $x^2y^2 - 9xy + 14$

15. $x^2y^2 + 6xy - 27$

16. $x^2y^2 - 3xy - 18$

Section 4

1. $2x^2 + 20x + 32$

2. $3x^2 + 30x + 63$

3. $x^3 - x^2 - 56x$

4. $3x^2 + 9x - 30$

5. $4x^2 - 4x - 48$

6. $2x^5 - 14x^4 + 24x^3$

7. $2x^2 + 24x + 64$

8. $4x^2y + 8xy - 12y$

9. $x^3 - 3x^2 - 28x$

10. $3x^2 - 6x - 45$

11. $3x^2 - 6x - 24$

12. $5x^2 - 20x + 20$

13. $3x^3 + 18x^2 + 27x$

14. $4x^3 + 8x^2 + 4x$

15. $7x^2 + 7x - 14$

Section 5

1. $2x^2 + 9x + 7$

2. $2x^2 + 13x + 15$

3. $3x^2 + 8x + 4$

4. $2x^2 - 9x - 5$

5. $3x^2 + 20x - 63$

6. $2x^2 - x - 6$

7. $8x^2 - 17x + 9$

8. $10x^2 + 17x + 3$

9. $2x^2 - 7x - 99$

10. $2x^2 + 7x - 72$

11. $4x^2 - 8x - 21$

12. $6x^2 - 11x - 10$

13. $2x^2 + 7x + 5$

14. $2x^2 + 7x + 3$

15. $3x^2 - 7x - 6$

16. $3x^2 + x - 2$

17. $8x^2 + x - 9$

18. $8x^2 + 6x - 27$

19. $12x^2 + 7x - 12$

20. $4x^2 - 3x - 7$

Section 6

1. $21x^2 - 48x - 45$

2. $12x^2 - 14x - 10$

3. $12x^2 - 14x - 6$

4. $12x^3 - 27x^2 - 27x$

5. $12x^3 - 62x^2 + 10x$

6. $4x^2 - 2x - 20$

7. $8x^2 + 46x - 12$

8. $8x^2 + 34x - 84$

9. $6x^2 - 2x - 60$

10. $10x^2 - 15x - 10$

11. $6x^3 - 16x^2 - 6x$

12. $28x^2 + 10x - 2$

13. $8x^2 - 20x + 12$

14. $30x^4 + 2x^3 - 4x^2$

15. $12x^2 + 15x - 18$

16. $18x^2 + 12x - 48$

Section 7

1. $9x^2 - 4$

2. $16x^2 - 25$

3. $16x^2 + 9$

4. $49x^2 - 36$

5. $81x^2 - 49$

6. $9x^2 + 4y^2$

7. $25x^2 + 16$

8. $25x^2 - 64$

9. $16x^2 - y^2$

10. $36x^2 - 49y^2$

11. $64x^2 - 25y^2$

12. $4x^2 + 49$

13. $x^2 - y^2$

14. $x^2 - 36y^2$

15. $x^2 + 16$

Section 8

1. $10 + x - 2x^2$

2. $8 - 2x - 3x^2$

3. $10 + x - 3x^2$

4. $12 + 16x - 3x^2$

5. $7 - 12x - 4x^2$

6. $16 + 22x - 3x^2$

7. $9 - 3x - 2x^2$

8. $15 - 4x - 4x^2$

9. $9 + 13x - 10x^2$

10. $12 - 11x - 5x^2$

11. $12 + 5x - 2x^2$

12. $18 - 9x - 2x^2$

13. $3 - 11x - 4x^2$

14. $14 + 3x - 2x^2$

15. $5 + 8x - 4x^2$

Section 9

1. $2x^3 + 3x^2 - 8x - 12$

2. $5x^3 - x^2 - 45x + 9$

3. $3x^3 - 2x^2 - 3x + 2$

4. $4x^3 + 12x^2 - 9x - 27$

5. $18x^3 + 9x^2 - 2x - 1$

6. $12x^3 - 16x^2 + 3x - 4$

7. $5x^3 - x^2 + 20x - 4$

8. $7x^3 + 3x^2 + 63x + 27$

9. $16x^3 - 12x^2 + 4x - 3$

10. $24x^3 - 4x^2 - 6x + 1$

11. $16x^3 - 4x^2 - 4x + 1$

12. $18x^3 - 27x^2 + 8x - 12$

13. $16x^3 - 32x^2 - x + 2$

14. $25x^3 - 25x^2 - 4x + 4$

15. $2x^3 + x^2 + 50x + 25$

16. $9x^3 - 4x^2 - 9x + 4$

Section 10

1. $x^3 - 8$

5. $x^3 + 1$

2. $x^3 - 27$

6. $x^3 - 64$

3. $x^3 + 8$

7. $x^3 - 1$

4. $x^3 + 64$

8. $x^3 + 27$

ANSWERS

Section 1

1. $4xy(9x + 2y^2)$

2. $-8x^8y^5(5y^2 + 2x)$

3. $5x^2y^2(3x - 5y)$

4. $-3x^2(xy + 3)$

5. $-5x^3y^4(3x^2 - 2y)$

6. $2x^3y(x - xy + y)$

7. $8xy^2(x^3 - 2y)$

8. $4x^2y(x - 3y + 2)$

9. $9(x - 3y - 9)$

10. $3x^2y(2x + 5y + 1)$

Section 2

1. $(x + 1)(x + 6)$

2. $(x + 4)(x + 2)$

3. $(x + 4)(x + 5)$

4. $(x + 3)(x + 10)$

5. $(x - 5)(x - 3)$

6. $(x - 7)(x - 2)$

7. $(x - 9)(x - 1)$

8. $(x - 3)^2$

9. $(x - 6)(x + 3)$

10. $(x - 6)(x + 5)$

11. $(x - 7)(x + 2)$

12. $(x - 12)(x - 4)$

13. $(x - 7)(x - 3)$

14. $(x - 7)(x + 3)$

15. $(x - 8)(x + 4)$

16. $(x - 9)(x - 7)$

17. $(x + 9)^2$

18. $(x - 6)^2$

19. $(x - 8)(x + 6)$

20. $(x + 7)(x - 6)$

Section 3

1. $(x + 5y)(x + 3y)$

2. $(x + 4y)(x + 2y)$

3. $(x - y)^2$

4. $(x - 6y)(x - 5y)$

5. $(x + 2y)(x + 5y)$

6. $(x + 5y)(x - 4y)$

7. $(x - 3y)(x - 4y)$

8. $(x - 3y)(x - 2y)$

9. $(xy + 4)(xy - 1)$

10. $(xy + 2)(xy - 1)$

11. $(xy - 3)^2$

12. $(xy - 3)(xy + 2)$

13. $(xy - 7)(xy + 3)$

14. $(xy - 2)(xy - 7)$

15. $(xy - 3)(xy + 9)$

16. $(xy + 3)(xy - 6)$

Section 4

1. $2(x + 8)(x + 2)$

2. $3(x + 7)(x + 3)$

3. $x(x - 8)(x + 7)$

4. $3(x + 5)(x - 2)$

5. $4(x - 4)(x + 3)$

6. $2x^3(x - 4)(x - 3)$

7. $2(x + 4)(x + 8)$

8. $4y(x + 3)(x - 1)$

9. $x(x - 7)(x + 4)$

10. $3(x - 5)(x + 3)$

11. $3(x - 4)(x + 2)$

12. $5(x - 2)^2$

13. $3x(x + 3)^2$

14. $4x(x + 1)^2$

15. $7(x + 2)(x - 1)$

Section 5

1. $(2x + 7)(x + 1)$

2. $(2x + 3)(x + 5)$

3. $(3x + 2)(x + 2)$

4. $(2x + 1)(x - 5)$

5. $(3x - 7)(x + 9)$

6. $(2x + 3)(x - 2)$

7. $(8x - 9)(x - 1)$

8. $(5x + 1)(2x + 3)$

9. $(2x + 11)(x - 9)$

10. $(2x - 9)(x + 8)$

11. $(2x - 7)(2x + 3)$

12. $(3x + 2)(2x - 5)$

13. $(2x + 5)(x + 1)$

14. $(2x + 1)(x + 3)$

15. $(3x + 2)(x - 3)$

16. $(3x - 2)(x + 1)$

17. $(8x + 9)(x - 1)$

18. $(4x + 9)(2x - 3)$

19. $(4x - 3)(3x + 4)$

20. $(4x - 7)(x + 1)$

Section 6

1. $3(7x + 5)(x - 3)$

2. $2(3x - 5)(2x + 1)$

3. $2(2x - 3)(3x + 1)$

4. $3x(4x + 3)(x - 3)$

5. $2x(6x - 1)(x - 5)$

6. $2(2x - 5)(x + 2)$

7. $2(4x - 1)(x + 6)$

8. $2(4x - 7)(x + 6)$

9. $2(3x - 10)(x + 3)$

10. $5(2x + 1)(x - 2)$

11. $2x(3x + 1)(x - 3)$

12. $2(7x - 1)(2x + 1)$

13. $4(2x - 3)(x - 1)$

14. $2x^2(5x + 2)(3x - 1)$

15. $3(4x - 3)(x + 2)$

16. $6(3x - 4)(x + 2)$

Section 7

1. $(3x - 2)(3x + 2)$

2. $(4x - 5)(4x + 5)$

3. prime

4. $(7x - 6)(7x + 6)$

5. $(9x - 7)(9x + 7)$

6. prime

7. prime

8. $(5x - 8)(5x + 8)$

9. $(4x - y)(4x + y)$

10. $(6x - 7y)(6x + 7y)$

11. $(8x - 5y)(8x + 5y)$

12. prime

13. $(x - y)(x + y)$

14. $(x - 6y)(x + 6y)$

15. prime

Section 8

1. $-1(2x - 5)(x + 2)$

2. $-1(3x - 4)(x + 2)$

3. $-1(3x + 5)(x - 2)$

4. $-1(3x + 2)(x - 6)$

5. $-1(2x + 7)(2x - 1)$

6. $-1(3x + 2)(x - 8)$

7. $-1(2x - 3)(x + 3)$

8. $-1(2x + 5)(2x - 3)$

9. $-1(5x - 9)(2x + 1)$

10. $-1(5x - 4)(x + 3)$

11. $-1(2x + 3)(x - 4)$

12. $-1(2x - 3)(x + 6)$

13. $-1(4x - 1)(x + 3)$

14. $-1(2x - 7)(x + 2)$

15. $-1(2x - 5)(2x + 1)$

Section 9

1. $(2x + 3)(x - 2)(x + 2)$

2. $(5x - 1)(x - 3)(x + 3)$

3. $(3x - 2)(x - 1)(x + 1)$

4. $(x + 3)(2x - 3)(2x + 3)$

5. $(2x + 1)(3x - 1)(3x + 1)$

6. $(3x - 4)(4x^2 + 1)$

7. $(5x - 1)(x^2 + 4)$

8. $(7x + 3)(x^2 + 9)$

9. $(4x - 3)(4x^2 + 1)$

10. $(6x - 1)(2x - 1)(2x + 1)$

11. $(4x - 1)(2x - 1)(2x + 1)$

12. $(2x - 3)(9x^2 + 4)$

13. $(x - 2)(4x - 1)(4x + 1)$

14. $(x - 1)(5x - 2)(5x + 2)$

15. $(2x + 1)(x^2 + 25)$

16. $(9x - 4)(x - 1)(x + 1)$

Section 10

1. $(x - 2)(x^2 + 2x + 4)$

2. $(x - 3)(x^2 + 3x + 9)$

3. $(x + 2)(x^2 - 2x + 4)$

4. $(x + 4)(x^2 - 4x + 16)$

5. $(x + 1)(x^2 - x + 1)$

6. $(x - 4)(x^2 + 4x + 16)$

7. $(x - 1)(x^2 + x + 1)$

8. $(x + 3)(x^2 - 3x + 9)$