
MATH 11009: Radicals

Example 1. Simplify: $\sqrt{24x^5y^6}$

SOLUTION.

$$\begin{aligned}\sqrt{24x^5y^6} &= \sqrt{4x^4y^6 \cdot 6x} \\ &= \boxed{2x^2y^3\sqrt{6x}}\end{aligned}$$

Example 2. Add/Subtract: $\sqrt{12} - 3\sqrt{8} + 4\sqrt{27}$

SOLUTION.

$$\begin{aligned}\sqrt{12} - 3\sqrt{8} + 4\sqrt{27} &= \sqrt{4 \cdot 3} - 3\sqrt{4 \cdot 2} + 4\sqrt{9 \cdot 3} \\ &= 2\sqrt{3} - 3(2)\sqrt{2} + 4(3)\sqrt{3} \\ &= 2\sqrt{3} - 6\sqrt{2} + 12\sqrt{3} \\ &= \boxed{14\sqrt{3} - 6\sqrt{2}}\end{aligned}$$

Example 3. Multiply: $(5\sqrt{x} + \sqrt{3})(3\sqrt{x} + \sqrt{3})$

SOLUTION. Use FOIL and note that $\sqrt{x} \cdot \sqrt{x} = x$:

$$\begin{aligned}(5\sqrt{x} + \sqrt{3})(3\sqrt{x} + \sqrt{3}) &= 15x + 5\sqrt{3x} + 3\sqrt{3x} + 3 \\ &= \boxed{15x + 8\sqrt{3x} + 3}\end{aligned}$$

Example 4. Multiply: $(\sqrt{x} - 7)^2$

SOLUTION. You must first write the binomial twice, then use FOIL:

$$\begin{aligned}(\sqrt{x} - 7)^2 &= (\sqrt{x} - 7)(\sqrt{x} - 7) \\ &= x - 7\sqrt{x} - 7\sqrt{x} + 49 \\ &= \boxed{x - 14\sqrt{x} + 49}\end{aligned}$$

Example 5. Simplify—rationalize the denominator where necessary: $\sqrt{\frac{5}{11}}$

SOLUTION.

$$\begin{aligned}\sqrt{\frac{5}{11}} &= \frac{\sqrt{5}}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} \\ &= \boxed{\frac{\sqrt{55}}{11}}\end{aligned}$$

Example 6. Rationalize the denominator and simplify: $\frac{10}{\sqrt{7} - 1}$

SOLUTION. Multiply the numerator and denominator by the conjugate of the denominator. Note that you must use FOIL on the denominator:

$$\begin{aligned}\frac{10}{\sqrt{7} - 1} \cdot \frac{(\sqrt{7} + 1)}{(\sqrt{7} + 1)} &= \frac{10(\sqrt{7} + 1)}{7 + \sqrt{7} - \sqrt{7} - 1} \\ &= \frac{10(\sqrt{7} + 1)}{6} \\ &= \boxed{\frac{5(\sqrt{7} + 1)}{3}}\end{aligned}$$

EXERCISES

Simplify:

1. $\sqrt{24x^2}$

7. $3x\sqrt{12x^2y^7}$

13. $\sqrt[3]{16x^4y^9}$

2. $\sqrt{18x^5y^7}$

8. $4y\sqrt{18x^5y^4}$

14. $\sqrt[3]{54x^5y^7}$

3. $\sqrt{32a^5b^{15}}$

9. $2x^2\sqrt{8x^2y^3}$

15. $\sqrt[3]{8x^6y^4}$

4. $\sqrt{40x^{11}y^7}$

10. $3y\sqrt{28x^3y^4}$

16. $\sqrt[3]{16x^5y^7}$

5. $2\sqrt{16a^2b^3}$

11. $5\sqrt{18xy^3}$

6. $4\sqrt{20a^4b^7}$

12. $2x\sqrt{48x^5y^2}$

17. $\sqrt[3]{40x^4y^8}$

Add/subtract and simplify:

18. $5\sqrt{4x} - 3\sqrt{9x}$

26. $3\sqrt{3x} + \sqrt{27x} - 8\sqrt{75x}$

19. $3\sqrt{3x^2} - 5\sqrt{27x^2}$

27. $x\sqrt{3y^2} - 2y\sqrt{12x^2} + xy\sqrt{3}$

20. $-2\sqrt{8x^2} + 5\sqrt{32x^2}$

28. $\sqrt{18} + \sqrt{8} - \sqrt{32}$

21. $5\sqrt{18} - 2\sqrt{75}$

29. $\sqrt{48} + \sqrt{20} - \sqrt{27} + 2\sqrt{20}$

22. $3x\sqrt{12x} - 5\sqrt{27x^3}$

30. $\sqrt{18x^3} + x\sqrt{32x} + \sqrt{50x^2}$

23. $8\sqrt{8} - 4\sqrt{32} - 9\sqrt{50}$

24. $-2\sqrt{3} + 5\sqrt{27} - 4\sqrt{45}$

31. $\sqrt{25} + \sqrt{12}$

25. $\sqrt{25x} - \sqrt{9x} + \sqrt{16x}$

32. $\sqrt{49} - \sqrt{90}$

Multiply:

33. $\sqrt{3a^2b^5}\sqrt{6ab^7}$

42. $(\sqrt{3x} + y)(\sqrt{3x} - y)$

34. $\sqrt{5x^3y}\sqrt{10x^2y}$

43. $(\sqrt{x} + 5)^2$

35. $\sqrt{2}(\sqrt{2} - \sqrt{3})$

44. $(2\sqrt{x} - 3)^2$

36. $3(\sqrt{12} - \sqrt{3})$

45. $(3 + \sqrt{2})(2 - \sqrt{3})$

37. $\sqrt{8}(\sqrt{2} - \sqrt{5})$

46. $(\sqrt{x} - 4)(\sqrt{x} + 4)$

38. $\sqrt{3a}(\sqrt{3a} - \sqrt{3b})$

47. $(\sqrt{x} + 3)^2$

39. $\sqrt{5x}(\sqrt{10x} - \sqrt{x})$

40. $(3\sqrt{x} - 2y)(5\sqrt{x} - 4y)$

48. $(\sqrt{x} + 4)(\sqrt{x} - 1)$

41. $(5\sqrt{x} + 2\sqrt{y})(3\sqrt{x} - \sqrt{y})$

49. $(4\sqrt{x} + \sqrt{3})^2$

Simplify—rationalize the denominator where necessary:

50. $\frac{8}{\sqrt{6}}$

55. $\sqrt{\frac{8}{9}}$

60. $\frac{18}{\sqrt{10}}$

51. $\frac{3}{\sqrt{3}}$

56. $\sqrt{\frac{2}{3}}$

61. $\frac{3}{\sqrt{12}}$

52. $\frac{1}{\sqrt{8}}$

57. $\sqrt{\frac{3}{5}}$

62. $\sqrt{\frac{5}{9}}$

53. $\frac{6}{\sqrt{12x}}$

58. $\frac{6}{\sqrt{3}}$

63. $\sqrt{\frac{8}{16}}$

54. $\sqrt{\frac{11}{4}}$

59. $\frac{4}{\sqrt{6}}$

64. $\sqrt{\frac{3}{5}}$

65.	$\sqrt{\frac{2}{7}}$	69.	$\frac{10}{\sqrt{7} + \sqrt{5}}$	73.	$\frac{12}{\sqrt{7} - 4}$
66.	$\frac{6}{\sqrt{7} + 3}$	70.	$\frac{6}{\sqrt{5} + 1}$	74.	$\frac{6}{\sqrt{5} + \sqrt{3}}$
67.	$\frac{2}{3 - \sqrt{2}}$	71.	$\frac{15}{\sqrt{3} - 6}$	75.	$\frac{5}{\sqrt{7} - \sqrt{3}}$
68.	$\frac{4}{1 + \sqrt{3}}$	72.	$\frac{11}{5 - \sqrt{3}}$		

ANSWERS:

1.	$2x\sqrt{6}$	12.	$8x^3y\sqrt{3x}$	23.	$-45\sqrt{2}$
2.	$3x^2y^3\sqrt{2xy}$	13.	$2xy^3\sqrt[3]{2x}$	24.	$13\sqrt{3} - 12\sqrt{5}$
3.	$4a^2b^7\sqrt{2ab}$	14.	$3xy^2\sqrt[3]{2x^2y}$	25.	$6\sqrt{x}$
4.	$2x^5y^3\sqrt{10xy}$	15.	$2x^2y\sqrt[3]{y}$	26.	$-34\sqrt{3x}$
5.	$8ab\sqrt{b}$	16.	$2xy^2\sqrt[3]{2x^2y}$	27.	$-2xy\sqrt{3}$
6.	$8a^2b^3\sqrt{5b}$	17.	$2xy^2\sqrt[3]{5xy^2}$	28.	$\sqrt{2}$
7.	$6x^2y^3\sqrt{3y}$	18.	\sqrt{x}	29.	$\sqrt{3} + 6\sqrt{5}$
8.	$12x^2y^3\sqrt{2x}$	19.	$-12x\sqrt{3}$	30.	$7x\sqrt{2x} + 5x\sqrt{2}$
9.	$4x^3y\sqrt{2y}$	20.	$16x\sqrt{2}$	31.	$5 + 2\sqrt{3}$
10.	$6xy^3\sqrt{7x}$	21.	$15\sqrt{2} - 10\sqrt{3}$	32.	$7 - 3\sqrt{10}$
11.	$15y\sqrt{2xy}$	22.	$-9x\sqrt{3x}$	33.	$3ab^6\sqrt{2a}$

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| 34. | $5x^2y\sqrt{2x}$ | 49. | $16x + 8\sqrt{3x} + 3$ | 63. | $\frac{\sqrt{2}}{2}$ |
| 35. | $2 - \sqrt{6}$ | 50. | $\frac{4\sqrt{6}}{3}$ | 64. | $\frac{\sqrt{15}}{5}$ |
| 36. | $3\sqrt{3}$ | 51. | $\sqrt{3}$ | 65. | $\frac{\sqrt{14}}{7}$ |
| 37. | $4 - 2\sqrt{10}$ | 52. | $\frac{\sqrt{2}}{4}$ | 66. | $-3(\sqrt{7} - 3)$ |
| 38. | $3a - 3\sqrt{ab}$ | 53. | $\frac{\sqrt{3x}}{x}$ | 67. | $\frac{2(3 + \sqrt{2})}{7}$ |
| 39. | $5x\sqrt{2} - x\sqrt{5}$ | 54. | $\frac{\sqrt{11}}{2}$ | 68. | $-2(1 - \sqrt{3})$ |
| 40. | $15x - 22y\sqrt{x} + 8y^2$ | 55. | $\frac{2\sqrt{2}}{3}$ | 69. | $5(\sqrt{7} - \sqrt{5})$ |
| 41. | $15x + \sqrt{xy} - 2y$ | 56. | $\frac{\sqrt{6}}{3}$ | 70. | $\frac{3(\sqrt{5} - 1)}{2}$ |
| 42. | $3x - y^2$ | 57. | $\frac{\sqrt{15}}{5}$ | 71. | $-\frac{5(\sqrt{3} + 6)}{11}$ |
| 43. | $x + 10\sqrt{x} + 25$ | 58. | $2\sqrt{3}$ | 72. | $\frac{5 + \sqrt{3}}{2}$ |
| 44. | $4x - 12\sqrt{x} + 9$ | 59. | $\frac{2\sqrt{6}}{3}$ | 73. | $-\frac{4(\sqrt{7} + 4)}{3}$ |
| 45. | $6 - 3\sqrt{3} + 2\sqrt{2} - \sqrt{6}$ | 60. | $\frac{9\sqrt{10}}{5}$ | 74. | $3(\sqrt{5} - \sqrt{3})$ |
| 46. | $x - 16$ | 61. | $\frac{\sqrt{3}}{2}$ | 75. | $\frac{5(\sqrt{7} + \sqrt{3})}{4}$ |
| 47. | $x + 6\sqrt{x} + 9$ | 62. | $\frac{\sqrt{5}}{3}$ | | |
| 48. | $x + 3\sqrt{x} - 4$ | | | | |