
MATH 11009: Rational Exponents

Definition. Rational (or fractional) exponents are defined as:

- $x^{1/n} = \sqrt[n]{x}$
- $x^{m/n} = \sqrt[n]{x^m}$ OR $x^{m/n} = (\sqrt[n]{x})^m$

Example 1. Write using rational exponents: $3\sqrt{x} - 2\sqrt[3]{x} + 8\sqrt[4]{x^3} + 7\sqrt[5]{x^2}$

SOLUTION.

$$3\sqrt{x} - 2\sqrt[3]{x} + 8\sqrt[4]{x^3} + 7\sqrt[5]{x^2} = \boxed{3x^{1/2} - 2x^{1/3} + 8x^{3/4} + 7x^{2/5}}$$

Example 2. Write using rational exponents: $\frac{5}{\sqrt[3]{x}} - \frac{2}{\sqrt[5]{x^3}} + 8\sqrt{x} + 4\sqrt{2x+1}$

SOLUTION.

$$\frac{5}{\sqrt[3]{x}} - \frac{2}{\sqrt[5]{x^3}} + 8\sqrt{x} + 4\sqrt{2x+1} = \boxed{5x^{-1/3} - 2x^{-3/5} + 8x^{1/2} + 4(2x+1)^{1/2}}$$

Example 3. Write using radicals: $5x^{1/2} + 3x^{2/3} + 7x^{-2/5} + 8x^{-1/3}$

SOLUTION.

$$\begin{aligned} 5x^{1/2} + 3x^{2/3} + 7x^{-2/5} + 8x^{-1/3} &= 5\sqrt{x} + 3\sqrt[3]{x^2} + 7\left(\frac{1}{x^{2/5}}\right) + 8\left(\frac{1}{x^{1/3}}\right) \\ &= \boxed{5\sqrt{x} + 3\sqrt[3]{x^2} + \frac{7}{\sqrt[5]{x^2}} + \frac{8}{\sqrt[3]{x}}} \end{aligned}$$

Example 4. Write using radicals: $\frac{2}{5}x^{1/3} + \frac{3}{7}x^{-1/2} + \frac{5}{8}x^{-2/3} - 5(7x-2)^{-1/2}$

SOLUTION.

$$\begin{aligned} \frac{2}{5}x^{1/3} + \frac{3}{7}x^{-1/2} + \frac{5}{8}x^{-2/3} - 5(7x-2)^{-1/2} &= \frac{2}{5}\sqrt[3]{x} + \frac{3}{7}\left(\frac{1}{x^{1/2}}\right) + \frac{5}{8}\left(\frac{1}{x^{2/3}}\right) - 5\left(\frac{1}{(7x-2)^{1/2}}\right) \\ &= \boxed{\frac{2}{5}\sqrt[3]{x} + \frac{3}{7\sqrt{x}} + \frac{5}{8\sqrt[3]{x^2}} - \frac{5}{\sqrt{7x-2}}} \end{aligned}$$

Example 5. Evaluate: $3(27)^{1/3} - 3(8)^{1/3}$

SOLUTION.

$$3(27)^{1/3} - 3(8)^{1/3} = 3\sqrt[3]{27} - 3\sqrt[3]{8} = 3(3) - 3(2) = 9 - 6 = \boxed{3}$$

Example 6. Evaluate: $\frac{4}{3}(9)^{3/2} - \frac{4}{3}(1)^{3/2}$

SOLUTION.

$$\begin{aligned}\frac{4}{3}(9)^{3/2} - \frac{4}{3}(1)^{3/2} &= \frac{4}{3}(\sqrt{9})^3 - \frac{4}{3}(\sqrt{1})^3 \\ &= \frac{4}{3}(3)^3 - \frac{4}{3}(1)^3 \\ &= \frac{4}{3}(27) - \frac{4}{3}(1) \\ &= \frac{108}{3} - \frac{4}{3} \\ &= \boxed{\frac{104}{3}}\end{aligned}$$

Example 7. Evaluate: $3(-1)^{-2/3} - 3(-8)^{-2/3}$

SOLUTION.

$$\begin{aligned}3(-1)^{-2/3} - 3(-8)^{-2/3} &= 3\left(\frac{1}{(\sqrt[3]{-1})^2}\right) - 3\left(\frac{1}{(\sqrt[3]{-8})^2}\right) \\ &= 3\left(\frac{1}{(-1)^2}\right) - 3\left(\frac{1}{(-2)^2}\right) \\ &= 3\left(\frac{1}{1}\right) - 3\left(\frac{1}{4}\right) \\ &= 3 - \frac{3}{4} \\ &= \frac{12}{4} - \frac{3}{4} \\ &= \boxed{\frac{9}{4}}\end{aligned}$$

EXERCISES

Write using (positive or negative) rational exponents:

1. $\sqrt[5]{x^2} + \sqrt[3]{x} - 2\sqrt{x}$
2. $5\sqrt[7]{x^4} + 8\sqrt{x} + \frac{2}{\sqrt[3]{x^2}} - \frac{5}{\sqrt[4]{x}}$
3. $\frac{3}{\sqrt{x}} + \frac{4}{\sqrt[3]{x}} - \frac{2}{\sqrt[4]{x}} - \frac{1}{\sqrt[5]{x}}$
4. $\frac{2}{3\sqrt[3]{x^2}} - \frac{4}{5\sqrt{x}} + \frac{3\sqrt{x}}{4} - \frac{5}{\sqrt[3]{x}}$
5. $3\sqrt{2x+5} - \sqrt[3]{3x-1} - \frac{2}{\sqrt{x+3}}$

Evaluate:

11. $4^{5/2}$
12. $16^{3/4}$
13. $9^{-3/2}$
14. $27^{-2/3}$
15. $(-1)^{2/5}$
16. $4(-8)^{-1/3}$
17. $5(4)^{3/2}$
18. $-3\left(\frac{8}{27}\right)^{2/3}$

Write using radicals:

6. $4x^{2/3} + 3x^{1/3} - 2x^{1/2}$
7. $5x^{1/2} + 2x^{2/3} + 3x^{3/4}$
8. $4x^{-2/3} + 2x^{-1/3} + 6x^{1/2}$
9. $\frac{3}{4}x^{-2/3} + \frac{3}{8}x^{-1/2} + \frac{7}{5}x^{1/3}$
10. $2x(x+4)^{1/2} + \frac{x^2(x+4)^{-1/2}}{2}$

19. $3(8)^{2/3} - 3(-1)^{2/3}$
20. $2(9)^{3/2} - 2(4)^{3/2}$
21. $\frac{2}{3}(4)^{-1/2} - \frac{2}{3}(1)^{-1/2}$
22. $5(-1)^{1/3} - 5(-8)^{1/3}$
23. $\frac{2}{5}(4)^{5/2} - \frac{2}{5}(1)^{5/2}$
24. $\frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}$
25. $-\frac{3}{2}(8)^{-2/3} + \frac{3}{2}(1)^{-2/3}$

ANSWERS

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|-----|--|-----|----------------|
| 1. | $x^{2/5} + x^{1/3} - 2x^{1/2}$ | 14. | $\frac{1}{9}$ |
| 2. | $5x^{4/7} + 8x^{1/2} + 2x^{-2/3} - 5x^{-1/4}$ | 15. | 1 |
| 3. | $3x^{-1/2} + 4x^{-1/3} - 2x^{-1/4} - x^{-1/5}$ | 16. | -2 |
| 4. | $\frac{2}{3}x^{-2/3} - \frac{4}{5}x^{-1/2} + \frac{3}{4}x^{1/2} - 5x^{-1/3}$ | 17. | 40 |
| 5. | $3(2x+5)^{1/2} - (3x-1)^{1/3} - 2(x+3)^{-1/2}$ | 18. | $-\frac{4}{3}$ |
| 6. | $4\sqrt[3]{x^2} + 3\sqrt[3]{x} - 2\sqrt{x}$ | 19. | 9 |
| 7. | $5\sqrt{x} + 2\sqrt[3]{x^2} + 3\sqrt[4]{x^3}$ | 20. | 38 |
| 8. | $\frac{4}{\sqrt[3]{x^2}} + \frac{2}{\sqrt[3]{x}} + 6\sqrt{x}$ | 21. | $-\frac{1}{3}$ |
| 9. | $\frac{3}{4\sqrt[3]{x^2}} + \frac{3}{8\sqrt{x}} + \frac{7\sqrt[3]{x}}{5}$ | 22. | 5 |
| 10. | $2x\sqrt{x+4} + \frac{x^2}{2\sqrt{x+4}}$ | 23. | $\frac{62}{5}$ |
| 11. | 32 | 24. | $\frac{14}{3}$ |
| 12. | 8 | 25. | $\frac{9}{8}$ |
| 13. | $\frac{1}{27}$ | | |