MATH 11009: EXAM 3 Review

• Section 3.1:
  – Identifying and evaluating exponential functions.
  – Identifying transformations in an exponential function
  – Application problems involving exponential functions

• Section 3.2:
  – Rewrite expression in either exponential or logarithmic form.
    \[ y = \log_b x \iff x = b^y \]
  – Evaluating logarithms
  – Application problems involving logarithmic functions.
  – Solving logarithmic equations.
  – Earthquake problems involving the Richter Scale.

• Section 3.3:
  – Use properties of logarithms to evaluate logs. (Note: these problems can also be solved without using properties.)
  – Rewrite an expression in either single log or expanded form using the Laws of Logarithms.
  – Use the change of base formula to calculate a logarithm to four decimal places.
    \[ \log_a x = \frac{\log_b x}{\log_b a} \]
  – Solve exponential equations.
  – Solve logarithmic equations. Remember to check “solution” to make sure you are not taking the logarithm of a negative number or zero.
  – Solve application problems involving exponential or logarithmic equations.

• Section 3.5:
  – Application problems involving present value and/or future value with compound interest and compound continuously interest
    \[ A(t) = P \left(1 + \frac{r}{n}\right)^{nt} \quad A(t) = Pe^{rt} \]
  – Application problems involving the amount of time for an investment to reach a certain level.
PRACTICE PROBLEMS
(These problems are for practice only. Please do not assume that only these types of problems are on the exam.)

1. Given the following exponential functions, determine if they show growth or decay.
   (a) \( y = 100(0.80)^n \)
   (b) \( y = 2^{-3x} \)
   (c) \( y = 0.5(6^k) \)
   (d) \( y = 5^x \)

2. Evaluate. Exact answers only
   (a) \( \log_4 \frac{1}{2} = \)
   (b) \( \log_{27} 9 = \)

3. The turtle population in local lake grows exponentially. The current population is 54 turtles and the relative growth rate is 21% per year. The turtle population \( P \) after \( t \) years is modeled using \( P = 54e^{0.21t} \).
   (a) Find the projected turtle population after 7 years. (Round to nearest whole unit).
   (b) Find the number of years required for the turtle population to reach 500. (Round to two decimal places).

4. Suppose the following continuous exponential function is used to model the population of a large city \( P = 800,000e^{-0.02t} \), where \( t \) is the number of years after 2003.
   (a) Does this model indicate that the population is increasing or decreasing
   (b) What is the population of the city in the year 2003?
   (c) Use this model to predict the population of the city in 2010. (Round answer to the nearest whole unit.)

5. An adoring aunt wants to guarantee that her niece will have enough money for college. How much should the aunt deposit into an account today with annual interest rate 4.97%, compounded quarterly, in order for the niece to have $45,000 for college 18 years from now?

6. If $22,450 is invested in an account earning 4\frac{1}{2}\% per year compounded monthly, determine the amount in the account at the end of 5 years. (Round answers to two decimal places.)
7. Rewrite the expression $\ln 20.09 = 3$ in exponential form.

8. Rewrite the expression $81^{-3/4} = \frac{1}{27}$ in logarithmic form.

9. Use the Change of Base Formula and a calculator to evaluate $\log_{15} 7$ correct to four decimal places.

10. Rewrite the following expression as a single logarithm.

$$\frac{1}{3} \left[ \ln y + 2 \ln(y + 4) \right] - 5 \ln(y - 1)$$

11. Use the Laws of Logarithms to rewrite the expression in a form with no logarithm of a product, quotient, or power where possible.

$$\log \frac{x^3 + 9}{x^4(2x - 1)^3}$$

12. Solve for $x$. Give BOTH the exact answer and a decimal approximation, accurate to four decimal places.

(a) $7 \left( 2 + 10^{4x} \right) = 35$

(b) $4^{5x+2} = 7$

(c) $2^{2x-3} + 3 = 8$

(d) $3(5 + e^{2x}) - 24 = 0$

13. Solve for $x$.

(a) $2 - \ln(3 - x) = 0$

(b) $\log_4(x + 2) + \log_4(x - 1) = 1$

(c) $\log(2x - 4) - 1 = 2$

(d) $\log(x + 4) - \log(x - 5) = 2$

14. If an earthquake has an intensity of 100,000 times $I_0$ what is the magnitude of the earthquake?

15. In 1989 the Richter scale reading for the San Francisco earthquake was 7.1. However, the largest earthquake to strike San Francisco occurred in 1906 with a Richter scale reading of 8.25. How many times more intense was the 1906 earthquake than the 1989 earthquake?
1. (a) decay  
   (b) decay  
   (c) growth  
   (d) growth

2. (a) $\frac{1}{2}$  
   (b) $\frac{2}{3}$

3. (a) 235 turtles  
   (b) 10.60 years

4. (a) decreasing  
   (b) 800,000  
   (c) 695,487

5. $18,496.37$

6. $28,102.82$

7. $20.09 = e^{3}$

8. $\log_{81} \frac{1}{27} = -\frac{3}{4}$

9. 0.7186

10. $\ln \frac{\sqrt[3]{y(y + 4)}}{(y - 1)^5}$

11. $\log(x^3 + 9) - 4 \log x - 3 \log(2x - 1)$

12. (a) $x = \frac{\log 3}{4}$  
    $x \approx 0.1193$
    (b) $x = \frac{\log 7}{5 \log 4} - \frac{2}{5}$  
    $x \approx -0.1193$
    (c) $x = \frac{\log 5}{2 \log 2} + \frac{3}{2}$  
    $x \approx 2.6610$
    (d) $x = \frac{\ln 3}{2}$  
    $x \approx 0.5493$

13. (a) $x = 3 - e^{2}$
    (b) $x = 2$
    (c) $x = 502$
    (d) $x = \frac{56}{11}$

14. $R = 5$

15. 14.13 times more intense.