MATH 11009: Exam #2 (Spring 2009)

1. A manufacturer of clarinets has monthly fixed costs of $\$13,750$ and variable costs of \$150 per clarinet, and it sells each clarinet for \$585.

   (a) Write the function that models the cost C from the production of x clarinets.

   (b) Write the function that models the profit P from the production and sale of x clarinets.

   (c) What is the profit if 75 clarinets are produced and sold?

2. Given $f(x) = 4x^2 - 24x + 7$.

   (a) Find the vertex of f.

   (b) Determine if the vertex is a maximum or minimum point. How do you know?

3. Explain how the graph of $y = -7\sqrt[3]{x+5}+2$ can be obtained from the graph of $y = \sqrt[3]{x}$.

4. Suppose the graph of $y = |x|$ is shifted right 8, reflected about the $x-$axis, vertically compressed by a factor of $\frac{6}{7}$, and shifted down 3. What is the equation of the new graph?

5. The profit from making and selling x units of a product is given by

   $$P(x) = -0.01x^2 + 20x - 500$$

   dollars. How many units should be produced and sold in order to make a profit of \$1400?

6. Solve each of the following. Show all work and simplify your answers.

   (a) $4x^2 - 9x - 10 = 30 - 5x^2$

   (b) $3(9x - 2)^2 - 24 = 0$

   (c) $6x^2 + 12x - 10 = 0$

7. If a ball is thrown into the air at 29.4 meters per second from a height of 60 meters, its height (in meters) is given by

$$h(t) = 60 + 29.4t - 9.8t^2,$$

where t is in seconds.

(a) When does the ball reach its maximum height?

(b) What is the maximum height of the ball?

8. Give an example of a graph of a function that is **NOT** one-to-one, and explain why it is not one-to-one.

9. Let $H(x) = 9(3-5x)^7+6$. Find nontrivial functions f and g such that $(f \circ g)(x) = H(x)$.

10. Find the inverse of $f(x) = \dfrac{7x+3}{2}$.

11. If $f(x) = 4x - 3$ and $g(x) = x^2 - 5x + 9$ find

(a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

12. Be able to determine the equation of any graph given. See handout from section 4.1 for examples.

ANSWERS

1. (a) $C(x) = 13,750 + 150x$
   (b) $P(x) = 435x - 13750$
   (c) $P(75) = \$18,875$

2. (a) $(3, -29)$
   (b) minimum since $a = 4 > 0$; opens up

3. reflect across x-axis; vertical stretch by factor of 7; left 5; up 2

4. $y = -\frac{6}{7}|x - 8| - 3$

5. 100 units or 1900 units

6. (a) $x = -\frac{5}{3},\ x = \frac{8}{3}$
   (b) $x = \frac{2 \pm 2\sqrt{2}}{9}$
   (c) $x = -1 \pm \frac{2\sqrt{6}}{3}$

7. (a) 1.5 seconds
   (b) 82.05 m

8. any example that does not pass the horizontal line test will work.

9. $f(x) = 9x^7 + 6, \quad g(x) = 3 - 5x$

10. $f^{-1}(x) = \frac{2x - 3}{7}$

11. (a) $4x^2 - 20x + 33$
    (b) $16x^2 - 44x + 33$

12. See section 4.1 handout for examples.