MATH 11009: Composition Functions Section 4.2

- **Combining Functions**:
  - **Sum:** $(f+g)(x) = f(x) + g(x)$
  - **Difference:** $(f-g)(x) = f(x) - g(x)$
  - **Product:** $(f \cdot g)(x) = f(x) \cdot g(x)$
  - **Quotient:** $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad (g(x) \neq 0)$
  - **Domain:** The domain of a sum, difference, and product of f and g consists of all real numbers of the input variable for which f and g are defined. The domain of the quotient function consists of all real numbers for which f and g are defined and $g(x) \neq 0$.

Example 1. Let $f(x) = x^3$ and $g(x) = \sqrt{x+3}$. Find the following:

(a) $(f+g)(x)$

(b) $(f-g)(x)$

(c) $(f \cdot g)(x)$

(d) $\left(\frac{f}{g}\right)(x)$

- **Total Cost Function:** denoted $C(x)$, is the total cost incurred in producing and selling x units of a certain commodity.
  - **Fixed costs:** include such things as rent, utilities, and equipment, and they remain constant regardless of how many units are produced.
  - **Variable costs:** are those directly related to the number of units produced.
  - $\boxed{C(x) = \text{fixed costs } + \text{ variable costs}}$

- **Revenue:** denoted $R(x)$, is the money received by a company from the sale of x units of a certain commodity. Hence,
$$\boxed{R(x) = x \cdot p,}$$
where x is the number of units sold and p is dollars per unit.

- **Profit Function:** denoted $P(x)$, is the found by subtracting the cost from the revenue.
$$\boxed{P(x) = R(x) - C(x).}$$

- **Average Cost Function:** denoted $\overline{C}(x)$, is the total cost divided by the number of units produced; hence,
$$\boxed{\overline{C}(x) = \frac{C(x)}{x}.}$$

Example 2. A manufacturer of computers has monthly fixed costs of $\$87,500$ and variable costs of \$87 per computer, and it sells the computer for \$295 per unit.

(a) Write the function that models the cost C from the production of x computers.

(b) Write the function that models the profit P from the production and sale of x computers.

(c) What is the profit if 700 computers are produced and sold in 1 month?

(d) Form the average cost function $\overline{C}(x)$. For what input value is $\overline{C}$ defined.

- **Composition of functions:** The composition function, f of g, is denoted by $f \circ g$ and defined by

$$\boxed{(f \circ g)(x) = f(g(x))}$$

- **Notes about composition functions:**
  - The domain of $f \circ g$ is the subset of the domain of g for which $f \circ g$ is defined.
  - Likewise, $(g \circ f)(x) = g(f(x))$. The domain of $g \circ f$ is the subset of the domain of f for which $g \circ f$ is defined.
  - In general, $(f \circ g)(x) \neq (g \circ f)(x)$

Example 3. For $f(x) = 3x - 2$; $g(x) = 2x^2 + 3x + 1$, find

(a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

(c) $(f \circ g)(2)$

Example 4. Let $f(x) = \sqrt{3-x}$; $g(x) = x - 5$. Find $(f \circ g)(4)$.

Example 5. Suppose $H(x) = 3(x-2)^2 + 4$. Find nontrivial functions f and g such that $F(x) = (f \circ g)(x)$.

Example 6. Suppose $H(x) = -5\sqrt{8x+3} + 2$. Find nontrivial functions f and g such that $F(x) = (f \circ g)(x)$.

Example 7. A man's shoe that is size x in Britain is size $d(x)$ in the United States, where $d(x) = x + 0.5$. A man's shoe that is size x in the United States is size $t(x)$ in Continental size, where $t(x) = x + 34.5$. Find a function that will convert British shoe size to Continental show size.