MATH 11009: Equations of lines
Section 1.4

- **Linear functions:** A function f is a linear function if it can be written as

$$f(x) = mx + b$$

  where m and b are constants. The graph of a linear function is always a straight line.

- **Horizontal lines:** Horizontal lines are given by equations of the type $y = b$ or $f(x) = b$.
- **Vertical lines:** Vertical lines are given by equations of the type $x = c$.
- **To find the equation of a line:** In order to find the equation of any line (that is not horizontal or vertical) we will always need two items: the slope and a point on the line. Once we have these two items, we need to use either the **slope-intercept form** or the **point-slope formula** to find the equation of the line. Although we have already discussed the slope-intercept form, it is stated here again for convenience.

Slope-intercept form: The slope-intercept form of an equation with slope m and $y-$intercept b is given by

$$y = mx + b.$$

Point-slope formula: The equation of the line with slope m and passing through (x_1, y_1) can be found using

$$y - y_1 = m(x - x_1).$$

Example 1. Find the equation of the line with slope $m = 3$ and that passes through $(-2, 5)$.

Example 2. Find the equation of the line with slope $m = -\frac{3}{4}$ and that passes through $(5, -2)$.

Example 3. Find the equation of the line passing through $(-2, 6)$ and $(4, -3)$.

- **Vertical Line:** A vertical line has the form $x = a$ where a is a constant and a is the $x-$coordinate of any point on the line. A vertical line has undefined slope.
- **Horizontal Line:** A horizontal line has the form $y = b$ where b is a constant and b is the $y-$coordinate of any point on the line. A horizontal line has zero slope.

Example 4. Find the equation of the line passing through $(3, 5)$ and $(3, -7)$.

Example 5. Find the equation of the horizontal line passing through $(-5, 9)$.

- **Average Rate of Change:** The average rate of change of $f(x)$ with respect to x over the interval from $x = a$ to $x = b$ (where $a < b$) is calculated as

$$\boxed{\text{average rate of change } = \frac{\text{change in } f(x) \text{ values}}{\text{corresponding changes in } x \text{ values}} = \frac{f(b) - f(a)}{b - a}}$$

  Note that the average rate of change between two points is the slope of the line joining those two points.

Example 6. For the function $f(x) = 2x^2 - 3x + 1$ find the average rate of change between $x = 1$ and $x = 3$.

Example 7. The composite SAT score for Beaufort county school district was 952 points in 2004, and the average rate of increase was 0.51 points per year. Write a linear function that models the SAT scores y as a function of x, the number of years after 2004.

Example 8. A company buys and retails baseball caps. The total cost function is linear. The total cost for 200 caps is $2680, and the total cost of 500 caps is $3530. Write the equation that models this cost function.

- **Parallel lines:** Parallel lines are two lines in the same plane that never intersect.
- **Perpendicular Lines:** Two lines are **perpendicular lines** if they intersect to form a 90° angle.

Parallel and Perpendicular Lines

- **Parallel lines** have the same slope. So, $m_1 = m_2$.
- **Perpendicular lines** have negative reciprocal slopes. In other words, $m_1 \cdot m_2 = -1$.

Example 9: Determine whether the following lines are parallel, perpendicular, or neither.

$$3x - 5y = 10 \quad \text{and} \quad 5x + 3y = 7$$

Example 10: Find the equation of the line that is parallel to $5x - 3y = 2$ and which passes through $(1, 3)$.

Example 11: Find the equation of the line that is perpendicular to $3x + 2y = 1$ and which passes through $(4, -2)$.

Difference Quotients: $\dfrac{f(x+h)-f(x)}{h}$

Example 12: Given $f(x) = 7x - 3$, find $\dfrac{f(x+h)-f(x)}{h}$.

Example 13: Given $f(x) = 2x^2 - 5x + 7$, find $\dfrac{f(x+h)-f(x)}{h}$.