MATH 11009: Exponential Functions Section 5.1

- **Exponential Function:** If $b > 0,\ b \neq 1$, then the function $f(x) = b^x$ is an exponential function. The constant b is called the **base** of the function and the variable x is the **exponent**.

- **The number e:** Many applications involve exponential functions with the base e. The number e is an irrational number where $e \approx 2.718281828$ (to nine decimal places).

Example 1. Determine which of the following functions are exponential functions.

(a) $y = 2e^{3x} + 7$

(b) $y = x^7 - 4x$

(c) $y = 4^{-x}$

Example 2. Given $f(x) = 4^{x-2} + 1$, find

(a) $f(3) =$

(b) $f(2) =$

(c) $f(0) =$

- **Exponential Growth Function:**

$$y = a(b^{kx}),\ b > 1,\ a > 0,\ k > 0.$$

  * There is no $x-$intercept.
  * $(0, a)$ is the $y-$intercept.
  * Domain is all real numbers.
  * Range is all $y > 0$.
  * Horizontal Asymptote is the $x-$axis $(y = 0)$.
    * If the values of y approach some finite number L as $|x|$ becomes very large, we say that the graph of $y = f(x)$ has a horizontal asymptote at $y = L$.
  * Increasing on the domain and concave up.

- **Exponential Decay Function:**

$$y = a(b^{-kx}),\ b > 1,\ a > 0,\ k > 0 \qquad \text{OR} \qquad y = a(c^{x}),\ 0 < c < 1,\ a > 0.$$

  * There is no $x-$intercept.
  * $(0, a)$ is the $y-$intercept.
  * Domain is all real numbers.
  * Range is all $y > 0$.
  * Horizontal Asymptote is the $x-$axis $(y = 0)$.
  * Decreasing on the domain and concave up.

Example 3. Use your knowledge of transformations to compare the graph of the following functions with the graph of $f(x) = 3^x$.

(a) $y = 3^{x-2} + 1$ (b) $y = -3^x - 5$

Example 4. At the end of an advertising campaign, weekly sales declined according to the equation $y = 10,000(3^{-0.05x})$ dollars, where x is the number of weeks after the campaign ended.

(a) Determine the sales at the end of the ad campaign.

(b) Determine the sales 8 weeks after the end of the campaign.

(c) How do we know, by inspecting the equation, that this function is decreasing?

Example 5. The population in a certain city was 800,000 in 2003, and its future size is predicted to be

$$P = 800,000e^{-0.02t}$$

people, where t is the number of years after 2003.

(a) Does this model indicate that the population is increasing or decreasing?

(b) Use this model to predict the population of the city in 2010.

(c) Use this model to predict the population of the city in 2020.

(d) What is the average rate of change in population between 2003 and 2010?