**MATH 11009: Functions**

- **Function**: A function is a rule or correspondence that assigns to each element of one set, called the domain, exactly one element of a second set, called the range. A function may be defined by a set of ordered pairs, a table, a graph, or an equation.

- **Domain**: The domain of a function is the set of all inputs. If \( x \) is any element in the domain, then \( x \) is called the independent variable.

- **Range**: The range of a function is the set of all outputs. If \( y \) represents an output of the function \( f \) from an input \( x \), then \( y \) is called the dependent variable and is denoted by \( f(x) \).

**Example 1**: Determine which of the following are examples of functions. For each function, determine the domain and range.

1. \{ (1,2), (3,6), (6,8), (9,2), (12,5) \}

2. \[
\begin{array}{c|ccccc}
 x & 1 & 2 & -5 & 2 & -4 \\
 y & -1 & 4 & 6 & 7 & 9 \\
\end{array}
\]

3. [Diagram showing mappings from \{1, 2, 3, -3\} to \{1, 4, 9\}]

4. [Diagram showing mappings from \{1, 2\} to \{1, 4\}]

5. \( x + y^2 = 9 \) where \( x \) is the input.
Example 2: For each of the following, determine whether or not the indicated relationship represents a function. For each function that is defined, give the domain and range.

1. The weight of a man in year $x$.

2. The daily profit $P$ (in dollars) from the sale of $x$ pounds of candy as shown in the following table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>$-100$</td>
<td>1050</td>
<td>1800</td>
<td>2050</td>
<td>1800</td>
<td>1050</td>
<td>$-200$</td>
</tr>
</tbody>
</table>

Example 3: Give an example of a graph that does not represent a function. What conclusion can you make?

Example 4: If $A(m) = 25 - 3m^2$, find

1. $A(2)$

2. $A(-3)$
Example 5: The following graph gives the number of men in the workforce (in millions) as a function $g$ of the year for selected years $t$ from 1890 to 1990.

1. Find and interpret $g(1940)$

2. What is the input $t$ if the output is $g(t) = 51,600,000$ men?

3. What can be said about the number of men in the workforce from 1890 to 1990?

4. What is the maximum number of men in the workforce during the period shown in the graph?

Finding Domain of a function: If the domain of a function is not provided, we assume that it includes all real numbers except those that give nonreal or undefined outputs. Examples of values that are not in the domain of a function are:

- values that result in a denominator of zero
- values that result in an even root of a negative number
Example 6: For each of the following functions determine the domain.

1. \( f(x) = \frac{x - 3}{x + 7} \)

2. \( g(x) = \frac{x}{x^2 - 5x + 6} \)

3. \( h(x) = \sqrt{x - 7} \)

Homework: pp 19–26; 1–23 odd, 29–45 odd, 51, 53