MATH 11009: Higher-Degree Polynomials Section 6.1

- **Polynomials:** Algebraic expressions containing a finite number of additions, subtractions, and multiplications of constants and nonnegative integer powers of variables are called **polynomials**. The general form of a polynomial in x is

  $$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

  where each coefficient $a_n, a_{n-1}, \ldots, a_1, a_0$ is a real number and each exponent $n, n-1, \ldots$ is a positive integer.

  * The **degree of a polynomial** is the highest power of x.
  * The **leading coefficient** is the coefficient of term with the highest power of x.

- In general, the graph of a polynomial with degree n has at most n $x-$intercepts.
- **Cubic Functions:** A **cubic function** in the variable x has an equation of the form

  $$f(x) = ax^3 + bx^2 + cx + d \qquad (a \neq 0)$$

  * A cubic function has at most 3 $x-$intercepts.
  * A cubic function will have either 0 or 2 turning points.
  * The end behavior of a cubic can be described as "one end opening up and one end opening down."
    - The cubic opens up on the right if the leading coefficient is positive.
    - The cubic opens down on the right if the leading coefficient is negative.

- **Quartic Functions:** A **quartic function** in the variable x has an equation of the form

  $$f(x) = ax^4 + bx^3 + cx^2 + dx + e \qquad (a \neq 0)$$

  * A quartic function has at most 4 $x-$intercepts.
  * A quartic function will have either 1 or 3 turning points.
  * The end behavior of a quartic can be described as "both ends up" or "both ends down"
    - The quartic has both ends up if the leading coefficient is positive.
    - The quartic has both ends down if the leading coefficient is negative.

Example 1. Use the equation of the polynomial function to state the degree and the leading coefficient, and describe the end behavior of the graph of the function.

(a) $f(x) = 4x^3 - 7x + 9x - 11$

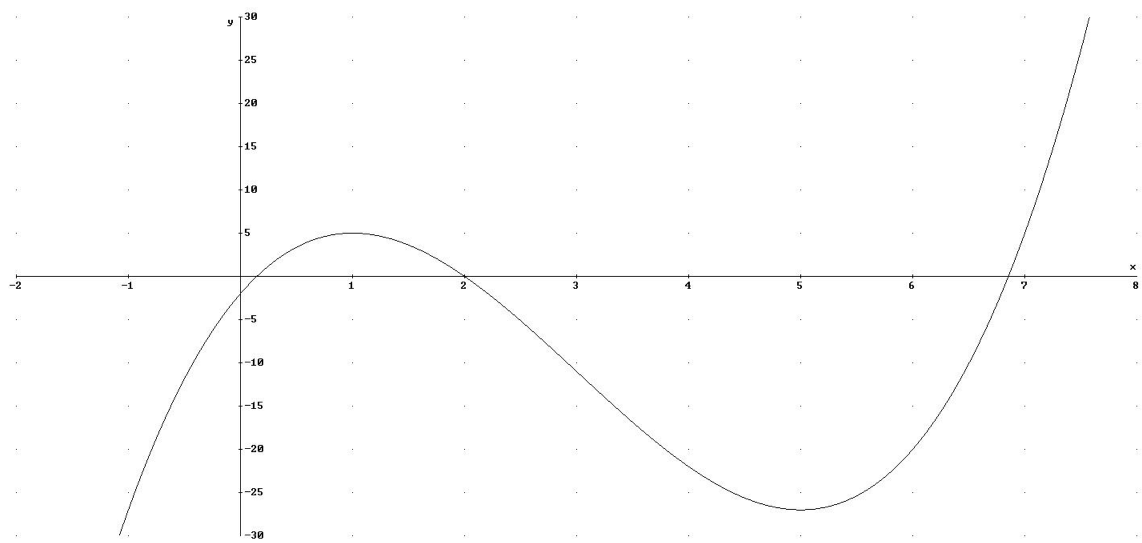
(b) $g(x) = -7x^4 + 9x^3 - 6x^2 + 5$

(c) $h(x) = -2x^3 + 8x - 4$

- **Higher Order Polynomials:**
  * A graph of a polynomial function of degree n has at most n $x-$intercepts.
  * The end behavior of a polynomial function with an odd degree can be described as "one end opening up and one end opening down."
  * The end behavior of a polynomial function with an even degree can be described as "both ends up" or "both ends down"
- **Zero**: If P is a polynomial and if c is a number such that $P(c) = 0$ then c is a **zero** of P. In fact, the following are all equivalent:
  * c is a zero of P
  * $(c, 0)$ is an $x-$intercept of the graph of P (when c is a real number)
  * $x - c$ is a factor of P
  * $x = c$ is a solution of the equation $P(x) = 0$
- **Even and Odd Multiplicity:** Let $k \geq 1$. If $(x - c)^k$ is a factor of a polynomial function P and $(x - c)^{k+1}$ is not a factor of P and:
  * k is odd, then the graph crosses the $x-$axis at $(c, 0)$.
  * k is even, then the graph is tangent to the $x-$axis at $(c, 0)$.

- **local extrema points:** The **local extrema points** of the graph of a function are the turning points of the graph. In particular, it is the points were the curve changes from decreasing to increasing or increasing to decreasing.
- **local minimum point:** A **local minimum point** is a point where the curve changes from decreasing to increasing.
- **local maximum point:** A **local maximum point** is a point where the curve changes from increasing to decreasing.
- **absolute maximum point:** The **absolute maximum point** is the highest point on the graph.
- **absolute minimum point:** The **absolute minimum point** is the lowest point on the graph.

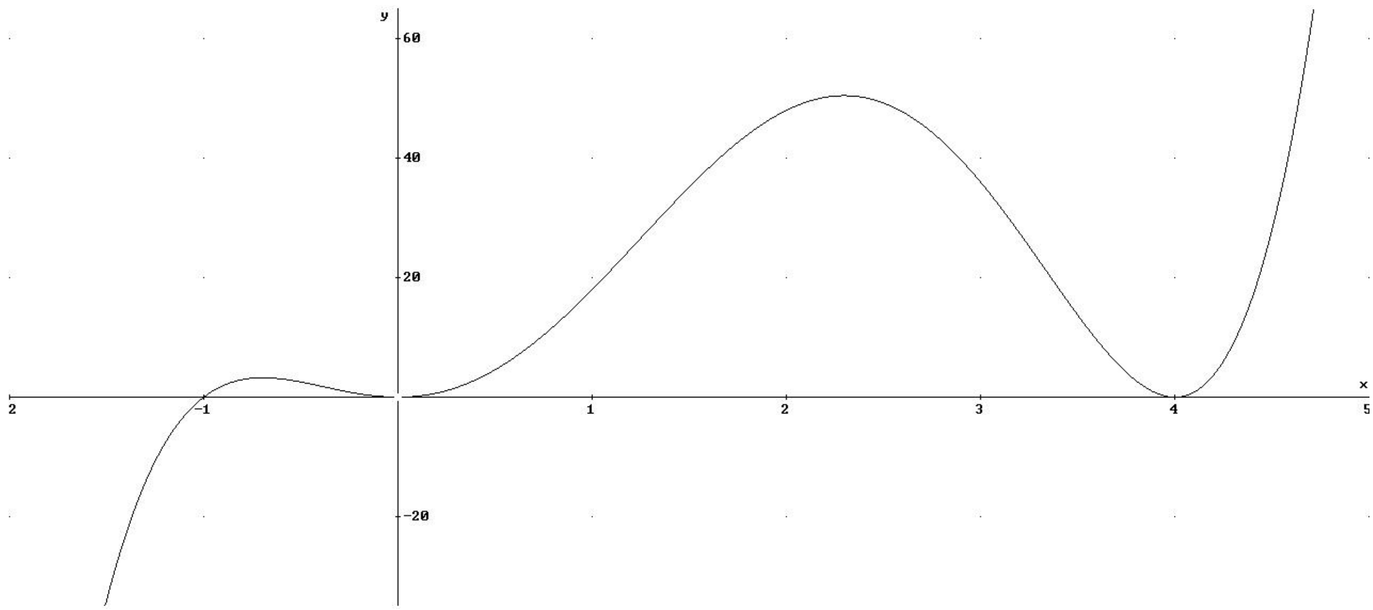
Example 2. Given below is the graph of g.



(a) Identify the local maximum(s).

(b) Identify the local minimum(s).

Example 3. Given below is the graph of f.



(a) Is the degree of f even or odd?

(b) Is the leading coefficient of f positive or negative?

(c) Determine the interval(s) where $f(x) \geq 0$.

(d) Determine the interval(s) where $f(x) < 0$.

(e) List the real zeros of f AND state whether each zero has even or odd multiplicity.