Example 1. Given $f(x) = 3x + 2$ and $g(x) = \frac{x - 2}{3}$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

- **Inverse Functions**: Functions $f$ and $g$ for which $f(g(x)) = x$ for all $x$ in the domain of $g$, and $g(f(x)) = x$ for all $x$ in the domain of $f$, are called inverse functions. In this case, we denote $g$ by $f^{-1}(x)$, read as $f$ inverse.

- The functions $f$ and $g$ are inverse functions, if whenever $(a, b)$ satisfies $y = f(x)$, the pair $(b, a)$ satisfies $y = g(x)$.

- Not all functions have an inverse. In fact, only one-to-one functions have an inverse.

- **One-to-One Functions**: A one-to-one function has exactly one output for each input and exactly one input for each output.

- **Horizontal Line Test**: A function is one-to-one if no horizontal line can intersect the graph of the function in more than one point.
Example 2. Determine if the function $f$ defined below has an inverse.

(a) $\{(3, 4), (6, 7), (9, 2), (4, 8)\}$

(b) $\{(1, 9), (2, 7), (3, 2), (4, 7)\}$

(c)

\[
\begin{array}{ccc}
1 & \rightarrow & 1 \\
2 & \rightarrow & 4 \\
3 & \rightarrow & 9 \\
-3 & \rightarrow & 9 \\
\end{array}
\]

(d) $f(x) = |x - 3| + 2$

(e) $f(x) = -\sqrt{x + 1} + 4$
• To find the inverse of a function that is defined by \( y = f(x) \):

1. Rewrite the equation replacing \( f(x) \) with \( y \).
2. Interchange \( x \) and \( y \) in the equation defining the function.
3. Solve the new equations for \( y \). If this equation cannot be solved uniquely for \( y \), the original function has no inverse function.
4. Replace \( y \) with \( f^{-1}(x) \).

**Example 3.** Find the inverse of \( f(x) = \frac{9x - 4}{2} \).

**Example 4.** Find the inverse of \( f(x) = \frac{1}{x - 3} \).
Example 5. Find the inverse of \{(3, 4), (6, 7), (9, 2), (4, 8)\}.

Example 6. If function \(h\) has an inverse and \(h^{-1}(9) = -1\), find \(h(-1)\).

- **Graphs of Inverse Functions**: The graphs of a function and its inverse are symmetric with respect to the line \(y = x\).

Example 7. The graph of \(f\) is given below. Sketch the graph of the \(f^{-1}\).