MATH 11009: Inverse Functions
Section 4.3

Example 1. Given $f(x) = 3x + 2$ and $g(x) = \dfrac{x-2}{3}$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

- **Inverse Functions:** Functions f and g for which $f(g(x)) = x$ for all x in the domain of g, and $g(f(x)) = x$ for all x in the domain of f, are called inverse functions. In this case, we denote g by $f^{-1}(x)$, read as f inverse.

- The functions f and g are inverse functions, if whenever (a, b) satisfies $y = f(x)$, the pair (b, a) satisfies $y = g(x)$.

- Not all functions have an inverse. In fact, only one-to-one functions have an inverse.

- **One-to-One Functions:** A one-to-one function has exactly one output for each input and exactly one input for each output.

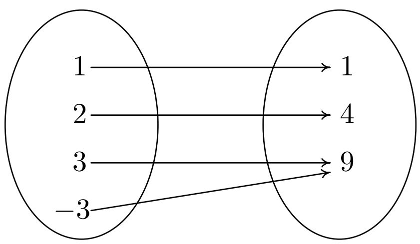
- **Horizontal Line Test:** A function is one-to-one if no horizontal line can intersect the graph of the function in more than one point.

Example 2. Determine if the function f defined below has an inverse.

(a) $\{(3,4),\ (6,7),\ (9,2),\ (4,8)\}$

(b) $\{(1,9),\ (2,7),\ (3,2),\ (4,7)\}$

(c)



(d) $f(x) = |x-3| + 2$

(e) $f(x) = -\sqrt{x+1} + 4$

- **To find the inverse of a function that is defined by** $y = f(x)$**:**
  1. Rewrite the equation replacing $f(x)$ with y.
  2. Interchange x and y in the equation defining the function.
  3. Solve the new equations for y. If this equation cannot be solved uniquely for y, the original function has no inverse function.
  4. Replace y with $f^{-1}(x)$.

Example 3. Find the inverse of $f(x) = \dfrac{9x - 4}{2}$.

Example 4. Find the inverse of $f(x) = \dfrac{1}{x - 3}$

Example 5. Find the inverse of $\{(3,4),\ (6,7),\ (9,2),\ (4,8)\}$.

Example 6. If function h has an inverse and $h^{-1}(9) = -1$, find $h(-1)$.

- **Graphs of Inverse Functions:** The graphs of a function and its inverse are symmetric with respect to the line $y = x$.

Example 7. The graph of f is given below. Sketch the graph of the f^{-1}