MATH 11009: Linear Functions Section 1.3

- **Linear Function:** A linear function is a function that can be written in the form $f(x) = ax + b$ or $y = ax + b$ where a and b are constants. The graph of a linear equation is straight line.
- **Slope:** The **slope** of a line measures its steepness. The slope, denoted by m, measures the vertical change and the horizontal change as we move along the line. The vertical change, also called the **rise**, is the difference between the y-coordinates. Therefore, rise is an up and down change. The horizontal change, also called the **run**, is the difference between the x-coordinates. Thus, run is a left and right change.
- **Slope formula:** If the coordinates of two points on the line are known then we can use the slope formula to find the slope of the line.

> The slope of the line through the points (x_1, y_1) and (x_2, y_2) is given by
> $$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}}$$

Note that it does not matter if you start with y_1 or y_2 in the numerator. However, you must start with its corresponding x in the denominator.

Example 1. Find the slope of the line passing through

(a) $(-3, 1)$ and $(7, -9)$.

(b) $\left(-\frac{1}{2}, 4\right)$ and $\left(\frac{7}{2}, \frac{7}{3}\right)$

The following table summarizes information concerning the slope of a line.

- If the slope is positive ($m > 0$), then the line slants up $\nearrow$
- If the slope is negative ($m < 0$), then the line slants down $\searrow$
- If the slope is zero ($m = 0$), then the line is horizontal $\rightarrow$
- If the slope is undefined, then the line is vertical $\uparrow$

- **Intercepts:** The points where a graph crosses or touches the $x-$axis and $y-$axis are called the $x-$**intercepts** and $y-$**intercepts**, respectively, of the graph.

HOW TO FIND THE INTERCEPTS ALGEBRAICALLY

To find the $y-$intercept of a graph of $y = f(x)$, set $x = 0$ in the equation and solve for y.

To find the $x-$intercept(s) of a graph of $y = f(x)$, set $y = 0$ in the equation and solve for x.

Example 2. Find the $x-$intercept and $y-$intercept of $5x - 3y = 2$. Write answers as ordered pairs.

- **Slope-Intercept form** of the line can be used to identify the slope and y-intercept.

> The **slope-intercept form** of an equation with slope m and $y-$intercept b is given by
> $$y = mx + b.$$

When identifying the slope and $y-$intercept using the slope-intercept form, remember to divide each term by the coefficient on y. The slope and $y-$intercept can only be identified once you have isolated y.

Example 3. Find the slope and $y-$intercept of $9x + 7y = 11$.

- **Constant Rate of Change:** The rate of change of the linear function $y = mx + b$ is the constant m, the slope of the graph of the function.

Example 4. Suppose the cost of a business property is \$1,920,000 and a company depreciates it with the straight-line method. Suppose v is the value of the property after x years, and the line representing the value as a function of years passes through the points $(10,\ \$1,310,000)$ and $(20,\ \$700,000)$.

(a) What is the slope of the line through these points? Interpret this value.

(b) What is the annual rate of change of the value of the property?

Example 5. A Business property is purchased with a promise to pay off a \$60,000 loan plus the \$16,500 interest on this loan by making 60 monthly payments of \$1,275. the amount of money, y, remaining to be paid on \$76,500 (the loan plus interest) is reduced by \$1,275 each month.

(a) Suppose x is the number of monthly payments made. Write a linear function to model this situation.

(b) Find the $x-$intercept and $y-$intercept of the graph of the linear function found in (a).

(c) Interpret the intercepts in the context of this problem situation.

(d) How should x and y be limited in this model so that they make sense in the applications.

(e) Use the intercepts and the results of part (d) to sketch the graph of the linear function.

Example 6. The total amount spent in the United States for wireless communication services S (in billion of dollars) can be modeled by

$$S(t) = 6.205 + 11.23t$$

where t is the number of years after 1995.

(a) Find the slope and interpret its meaning.

(b) Find the vertical intercept and interpret its meaning.