MATH 11009: Logarithmic Functions Section 5.2

- **Logarithmic Function:** For $x > 0,\ b > 0$, and $b \neq 1$, the logarithmic function to base b is $y = \log_b x$ which is defined by $x = b^y$.

  * Note that $y = \log_b x$ and $x = b^y$ are two different forms of the same equation. We say $y = \log_b x$ is the **logarithmic form** of the equation and $x = b^y$ is the **exponential form** of the equation.

Example 1. Rewrite $64 = 4^3$ in logarithmic form.

Example 2. Rewrite $-3 = \log_{1/2} 8$ in exponential form.

Example 3. Evaluate each logarithm.

(a) $\log_9 81$

(b) $\log_4 2$

(c) $\log_3 \dfrac{1}{81}$

- **Notes on the graph of a logarithmic function:**
  - The $x-$intercept is $(1, 0)$.
  - There is no $y-$intercept.
  - The domain is $x > 0$.
  - The range is all real numbers.
  - There is a vertical asymptote at $x = 0$ (the $y-$axis).
  - If $b > 1$, then the graph is increasing.
  - If $0 < b < 1$, then the graph is decreasing.

- **Common Logarithm:** Logarithms with base $b = 10$ are called **common logarithms**. In this case, we write $\log x$ instead of $\log_{10} x$.

- **Natural Logarithm:** Logarithms with base $b = e$ are called **natural logarithms**. In this case, we write $\ln x$ instead of $\log_e x$.

Example 4. Use a calculator to evaluate the following logarithms to four decimal places.

(a) $\log 93$ (b) $\ln 5$

Example 5. The population of Japan for the years 1984–2000 is approximated by the logarithmic function

$$y = 114.198 + 4.175 \ln x$$

million people, with x equal to the number of years after 1980. According to the model, what is the estimated population in 1986? in 2000?

Example 6. Solve $\log_4 x = -2$.

Example 7. Solve $4 + 3\log x = 10$.

- **Richter Scale:** The Richter Scale gives the magnitude R of an earthquake using the formula
$$R = \log\left(\frac{I}{I_0}\right)$$
where I is the intensity of the earthquake and I_0 is a certain minimum intensity used for comparison.

    * **Comparing two earthquake measurements:** If the difference of the Richter scale measurements of two earthquakes is the positive number d, the intensity of the larger earthquake is 10^d times more than that of the smaller earthquake.

Example 8. If an earthquake has an intensity of 10,000 times I_0, what is the magnitude of the earthquake?

Example 9. An earthquake that measured 9.0 on the Richter scale occurred in the Indian Ocean in December 2004, causing a devastating tsunami that killed thousands of people. Express the intensity of this earthquake in terms of I_0.

Example 10. If an earthquake measures 7.0 on the Richter scale, give the intensity of this earthquake in terms of I_0. How much more is the intensity of the earthquake in example 9 than the one with Richter scale measurement of 7.0?