MATH 11009: Modeling Linear Functions
Section 2.2

- **Linear Regression**: A procedure to determine the equation of the line that is the “best fit” for a set of data points. This method defines the “best fit” line as the line for which the sum of the square of the vertical distances from the data points to the line is a minimum; also called the **Least Squares Method**.

- **Discrete function**: A **discrete function** is a function that has a finite number of inputs. (NOTE: If a continuous function is used to model a discrete function then we cannot use it to estimate the output of every real number input.)

- **Continuous function**: A **continuous function** is a function whose inputs can be any real number or any real number between two specified values.

**Example 1.** Determine whether each of the following is an example of a discrete function or a continuous function.

(a) Suppose a pizza shop charges $2 for a small pizza, and $0.75 for each topping, with up to 8 toppings available. The total price $P$ of the pizza is determined by the number of toppings $t$. Hence, $P(t) = 2 + 0.75t$.

(b) The percentage of unmarried women who get married in any particular year is given by $y = -0.0762x + 8.5284$, where $x$ is the number of years after 1950.

(c) Suppose a ball thrown into the air has its height (in feet) given by the function

$$ h(t) = 6 + 96t - 16t^2 $$

where $t$ is the measured in seconds.

- **Exact and Approximate Linear Models**:

  - If the first differences of data outputs are constant for uniform inputs, the rate of change is constant and a linear function can be found that fits the data exactly.
  - If the first differences are “nearly constant,” a linear function can be found that is an approximate fit for the data.
Example 2. The following data is recorded:

<table>
<thead>
<tr>
<th>(x)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
</tr>
</tbody>
</table>

(a) Create a scatter plot for the set of data.

(b) Can the scatter plot above be fit exactly by a linear function, only approximately by a linear function, or neither? How do you know?

Example 3. The following data is recorded:

<table>
<thead>
<tr>
<th>(x)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>1</td>
<td>9</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

(a) Create a scatter plot for the set of data.

(b) Can the scatter plot above be fit exactly by a linear function, only approximately by a linear function, or neither?
Example 4. Determine which of the equations, \( y = 2.3x + 4 \) or \( y = 2.1x + 6 \), is a better fit for the data points 

\[(20, 48), \ (30, 72), \ (40, 97), \ (50, 120), \ (60, 142)\]

Example 5. The average price \( p \) of iceberg lettuce, in cents per pound, in US cities between February and June 1999 is a function of the month \( x \) in which the average price was determined. Does the point \((5.75, \ p(5.75))\) have a valid meaning in this situation? If so, interpret the coordinates of the point. If not, explain why.
Example 6. The number of women in the workforce for selected years from 1890 to 1990 is shown in the following figure.

(a) Does the scatter plot in the figure define the number of working women as a discrete or continuous function of the year?

(b) Would the data in the scatter plot be better modeled by a linear function or a nonlinear function? Why or why not?