
MATH 11009: Additional Equations & Inequalities

Section 4.4

- **Radical equations:** A **radical equation** is an equation in which variables appear in one or more radicands. To solve a radical equation we can use the following principle:

For any positive integer n , if $a = b$ is true, then $a^n = b^n$ is true.

- **CAUTION:** The converse of the above statement is not true. For example, $(-4)^2 = (4)^2$ is true, but $-4 \neq 4$.

Solving an equation involving a radical:

1. Isolate a single radical on one side of the equation.
2. Raise both *sides* of the equation to the same power that is the same as the index on the radical.
3. If a radical remains, repeat steps 1 and 2.
4. Solve the resulting equation.
5. All solutions must be checked in the original equation, and only those that satisfy the original equation are actual solutions.

Example 1: Solve: $\sqrt{x+3} - 1 = x$

Example 2: Solve: $\sqrt{x+7} - \sqrt{2x-3} = 2$

Example 3. Solve: $x^{2/3} - 4 = 0$

Example 4. Solve: $x^{3/2} - 8 = 0$

Guidelines for solving nonlinear inequalities:

1. Move all terms to one side of the inequality.
2. Factor completely.
3. Set each factor equal to zero. These numbers divide the real number line into intervals.
4. Make a sign chart by dividing the real number line into the intervals found in Step 3.
5. In each interval, use a test value to determine the sign (positive or negative) of the product or quotient in that interval.
6. Determine the solution of the inequality by examining the sign values. Be sure to check whether the inequality is satisfied at some or all of the endpoints.

Example 5. Solve: $x^2 - x - 6 < 0$

Example 6. Solve: $x^3 < 4x^2$

Example 7. Solve: $x^3 + x^2 - 4x - 4 \geq 0$

Absolute value inequalities: Let $c > 0$. Then

- $|ax + b| \leq c$ is equivalent to $-c \leq ax + b \leq c$
- $|ax + b| \geq c$ is equivalent to $ax + b \geq c$ or $ax + b \leq -c$

Example 8. Solve: $|2x - 3| \leq 7$

Example 9. Solve: $|5x + 3| > 9$

Example 10. Solve: $|9 - 4x| \geq 8$

Exercises

Solve for x :

1. $\sqrt{7-5x} = 8$

11. $\sqrt{3x+6} = x+2$

2. $\sqrt{5x+2} = 6$

12. $x-1-\sqrt{4x-7} = 0$

3. $3\sqrt{8x-1} - 6 = 0$

13. $\sqrt{x+1} - 2x - 1 = 0$

4. $4\sqrt{7x+3} = 12$

14. $x+2-\sqrt{x+5} = 0$

5. $3\sqrt{2x-1} + 5 = 14$

15. $\sqrt{2x+8} = \sqrt{4x+9} + 1$

6. $8\sqrt{9x+2} - 10 = 6$

16. $\sqrt{11x+31} - \sqrt{5x+10} = 3$

7. $4\sqrt{5x} - 5 = 0$

17. $x^{3/2} - 1 = 0$

8. $3\sqrt{2x+1} - 4 = 1$

18. $x^{2/3} - 2 = 0$

9. $\sqrt{2x+8} = \sqrt{4x+9}$

19. $x^{2/3} - 3 = 0$

10. $\sqrt{2x-3} = x-3$

20. $x^{2/5} + 2 = 0$

ANSWERS

1. $x = -57/5$

2. $x = 34/5$

3. $x = 5/8$

4. $x = 6/7$

5. $x = 5$

6. $x = 2/9$

7. $x = 5/16$

8. $x = 8/9$

9. $x = -1/2$

10. $x = 6$

11. $x = 1, -2$

12. $x = 2, 4$

13. $x = 0$

14. $x = \frac{-3}{2} + \frac{\sqrt{13}}{2}$

15. $x = -2$

16. $x = -2, 3$

17. $x = 1$

18. $x = \pm 2\sqrt{2}$

19. $x = \pm 3\sqrt{3}$

20. No solution