MATH 11009: More Polynomial Equations Section 6.4

- **Dividend**: The number or expression you are dividing into.
- **Divisor**: The number or expression you are dividing by.
- **Synthetic division**: **Synthetic division** is a quick method of dividing polynomials when the divisor is of the form $x - c$ or $x + c$.
- **Zero**: If P is a polynomial and if c is a number such that $P(c) = 0$ then c is a **zero** of P. In fact, the following are all equivalent:

  * c is a zero of P
  * $(c, 0)$ is an $x-$intercept of the graph of P (when c is a real number)
  * $x - c$ is a factor of P
  * $x = c$ is a solution of the equation $P(x) = 0$

- **Steps for synthetic division to divide $P(x)$ by $x - c$**: Synthetic division will consist of three rows.

  1. Write the c and the coefficients of the dividend in descending order in the first row. If any x terms are missing, place a zero in its place.
  2. Bring the leading coefficient in the top row down to the bottom (third) row.
  3. Next, multiply the first number in the bottom row by c and place this product in the second row under the next coefficient and add these two terms together.
  4. Continue this process until you reach the last column.
  5. The numbers in the bottom row are the coefficients of the quotient, with the last number the remainder. The quotient will have one degree less than the dividend.
  6. NOTE: If the remainder is zero, then $x - c$ is a factor of the polynomial, and the polynomial can be written as the product of the divisor $x - c$ and the quotient.

Example 1. Use synthetic division to find the quotient and remainder.

$$\left(x^4 + 2x^3 - 3x^2 + 1\right) \div (x + 4)$$

Example 2. Use synthetic division to find the quotient and remainder.

$$\frac{2x^4 - 3x^3 - 10x^2 + 5}{x - 3}$$

- **Remainder Theorem**: If a polynomial $P(x)$ is divided by $x - c$, then the remainder is $P(c)$. This gives us another way to evaluate a polynomial at c.
- **Factor Theorem**: c is a zero of $P(x)$ if and only if $x - c$ is a factor of $P(x)$.

Example 3. Determine whether the second polynomial is a factor of the first polynomial.

$$2x^4 + 5x^3 - 6x - 4 \qquad\qquad x + 2$$

Example 4. One or more solutions of the a polynomial equation are given. Use synthetic division to find any remaining solutions.

$$2x^4 - 17x^3 + 51x^2 - 63x + 27 = 0; \qquad 3,\ 1$$

- **Rational Zeros Theorem**: If the polynomial

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

has integer coefficients, then every rational zero of f is of the form $\frac{p}{q}$ where

$$\begin{aligned} &p \text{ is a factor of the constant term } a_0, \\ \text{and } &q \text{ is a factor of the leading coefficient } a_n. \end{aligned}$$

- The Rational Zeros Theorem does NOT list irrational zeros. These will need to be found using other means like the quadratic formula.

Example 5. List all POSSIBLE rational zeros of $P(x) = 2x^3 + 15x^2 + 22x - 15$.

- **Steps for finding the real zeros of a polynomial**:

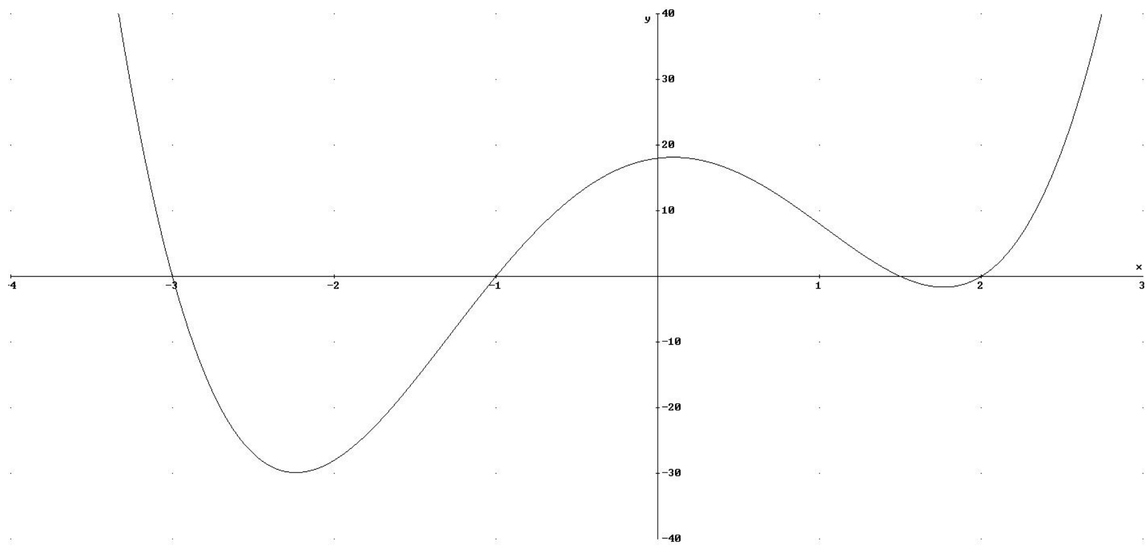
  1. List all *possible* rational zeros using the Rational Zeros Theorem.
  2. Use synthetic division to test the polynomial at each of the possible rational zeros that you found in step 1. (Remember that c is a zero when the remainder is zero.)
  3. Repeat step 2 until you reach a quotient that is a quadratic or factors easily. Use the quadratic formula or factoring to find the remaining zeros.

Example 6. Find all real zeros of

$$f(x) = 4x^3 + 3x^2 - 9x + 2$$

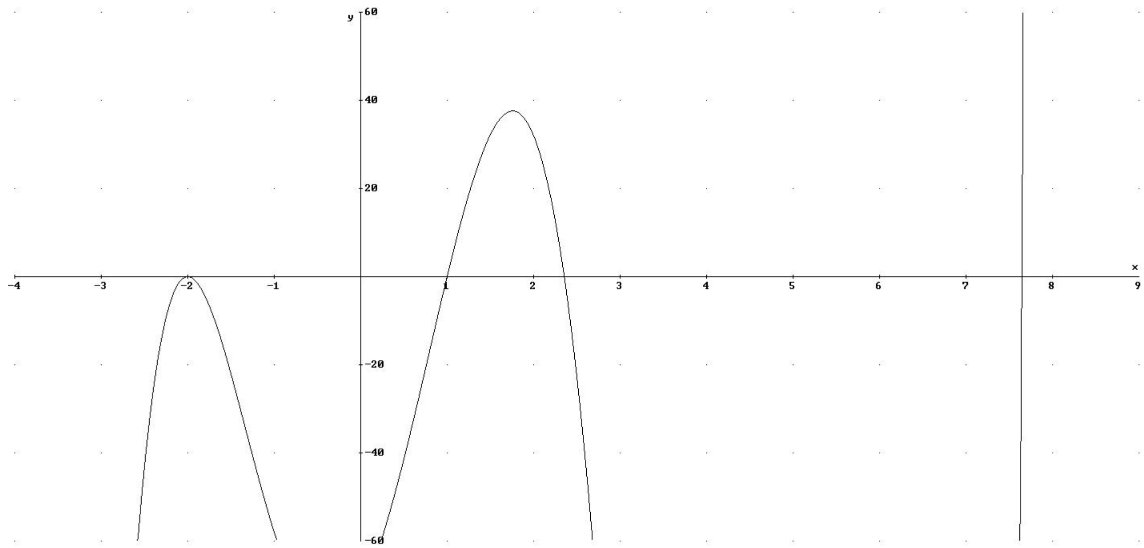
Example 7. Find all real zeros of

$$f(x) = 2x^4 + x^3 - 16x^2 + 3x + 18$$



Example 8. Find all real zeros of

$$f(x) = x^5 - 7x^4 - 12x^3 + 50x^2 + 40x - 72$$



Example 9. The profit function for a product is given by

$$P(x) = -x^3 + 98x^2 - 700x - 1800$$

dollars, where x is the number of units produced and sold. It can be shown that break-even occurs when 10 units are produced and sold. Find another number of units other than 10 that given break-even for the product.