
MATH 11009: Solving Polynomial Equations

Section 6.3

- Recall the following properties from Chapter 3:
 - **Zero Product Property:** For real numbers a and b , the product $ab = 0$ if and only if either $a = 0$ or $b = 0$ or both a and b are zero. (Note that this can be extended to any number of factors.)
 - **Square Root Method:** The solutions of the quadratic equation $x^2 = C$ are $x = \sqrt{C}$ and $x = -\sqrt{C}$. These solutions can be combined as $x = \pm\sqrt{C}$. Note that this method can also be used to solve a quadratic equation of the form $(ax+b)^2 = C$.
- **The Root Method:** The real solutions of the equation $x^n = C$ are found by taking the n th root of both sides:

$$x = \sqrt[n]{C} \quad \text{if } n \text{ is odd} \qquad x = \pm\sqrt[n]{C} \quad \text{if } n \text{ is even and } C \geq 0.$$

Note that this method can also be used to solve an equation of the form $(ax+b)^n = C$.

Example 1. Solve: $12x^3 + 10x^2 - 8x = 0$

Example 2. Solve: $x^3 + 5x^2 - 7x - 35 = 0$

Example 3. Solve: $2x^4 - 162 = 0$

Example 4. Solve: $4(2x - 1)^3 = 32$

Example 5. Solve: $3(3x + 2)^4 = 21$

Example 6. The revenue from the sale of a product is given by the function

$$R(x) = 12,000x - 0.003x^3.$$

Find the number of units that must be sold to give zero revenue.

Example 7. Suppose \$5,000 is invested in an account for 4 years at a rate of $r\%$ compounded annually. Find r (as a percent) if the future value of the account is \$10,368.

Recall the following concepts:

- **Zero:** If P is a polynomial and if c is a number such that $P(c) = 0$ then c is a **zero** of P . In fact, the following are all equivalent:
 - * c is a zero of P
 - * $(c, 0)$ is an x -intercept of the graph of P (when c is a real number)
 - * $x - c$ is a factor of P
 - * $x = c$ is a solution of the equation $P(x) = 0$

- **Even and Odd Multiplicity:** Let $k \geq 1$. If $(x - c)^k$ is a factor of a polynomial function P and $(x - c)^{k+1}$ is not a factor of P and:
 - * k is odd, then the graph crosses the x -axis at $(c, 0)$.
 - * k is even, then the graph is tangent to the x -axis at $(c, 0)$.

Example 8. Below is the graph of $f(x) = x^4 - 5x^3 - 3x^2 + 13x + 10$. Use the graph of f to (a) solve $f(x) = 0$, and (b) find the factorization of $f(x)$.

