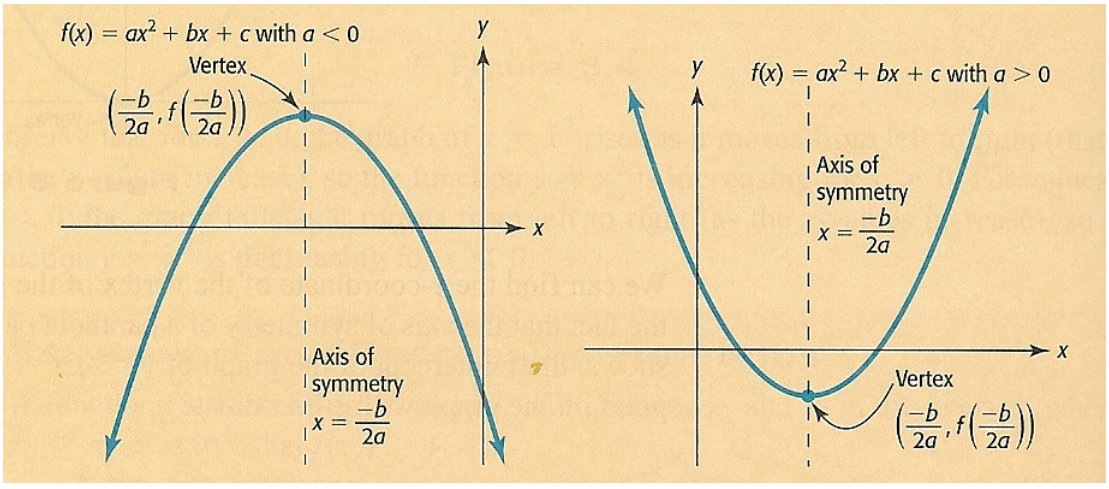
MATH 11009: Quadratic Functions
Section 3.1

- **Quadratic Function:** A **quadratic function** is a function that can be written in the form
$$f(x) = ax^2 + bx + c$$
where a, b, and c are real numbers and $a \neq 0$.

- **Notes on the graph of a quadratic function:**
  - The graph of a quadratic function is a parabola.
  - The "turning point" of the parabola is called the **vertex**. The vertex of the parabola is located at
$$\boxed{\left(-\frac{b}{2a},\ f\left(-\frac{b}{2a}\right) \right)}$$
  - The vertical line through the vertex is called the **axis of symmetry** because this line divides the graph into two halves that are reflections of each other. The axis of symmetry of the parabola has the equation
$$\boxed{x = -\frac{b}{2a}}$$
  - The parabola opens up, if $a > 0$. In this case, we say that the parabola is **concave up** and the vertex is a **minimum point**.
  - The parabola opens down, if $a < 0$. In this case, we say that the parabola is **concave down** and the vertex is a **maximum point**.



Example 1. Let $f(x) = 4x^2 - 16x + 5$.

(a) Find the axis of symmetry of the graph of this function.

(b) Find the vertex of the graph of this function.

(c) Determine if the vertex represents a maximum or minimum point. How do you know?

(d) Sketch the graph of this function.

Example 2. If a ball is thrown upward at 39.2 meters per second from the top of a building that is 30 meters high, the height of the ball can be modeled by $S(t) = -9.8t^2 + 39.2t + 30$ meters, where t is the number of seconds after the ball is thrown.

(a) Find the coordinates of the vertex of the graph of this quadratic function.

(b) Explain the meaning of the coordinates of the vertex for this function.

(c) Over what time interval is the function increasing? What does this mean in relation to the ball?

Example 3. The profit for a product can be described by the function

$$P(x) = -0.4x^2 + 840x - 75.6$$

million dollars, where x is the number of units produced and sold.

(a) To maximize profit, how many units must be produced and sold?

(b) What is the maximum possible profit?

- **Quadratic Functions in standard form:** The graph of the function

$$\boxed{f(x) = a(x-h)^2 + k}$$

is a parabola with its vertex at the point (h, k).

  - The axis of symmetry of the parabola is the vertical line $x = h$.
  - If $a > 0$, then the parabola opens up and the vertex is a minimum point.
  - If $a < 0$, then the parabola opens down and the vertex is a maximum point.

Example 4. Let $y = 3(x+2)^2 + 7$.

(a) Determine the axis of symmetry of the graph of this function.

(b) Determine the vertex of the graph of this function.

(c) Is the graph of this function concave up or concave down?

(d) Does this function have a maximum or minimum? How do you know?

(e) Sketch a graph of this function.