MATH 11009: Solving Quadratic Equations Section 3.2

- **Quadratic Equations:** A **quadratic equation** is an equation of the form

$$ax^2 + bx + c = 0, \quad a \neq 0$$

where a, b, and c are real numbers.

- **Quadratic Functions:** A **quadratic function** f is a function that can be written in the form

$$f(x) = ax^2 + bx + c, \quad a \neq 0$$

where a, b, and c are real numbers.

- **Zero Product Property:** For real numbers a and b, the product $ab = 0$ if and only if either $a = 0$ or $b = 0$ or both a and b are zero. (Note that this property can be extended to more than two factors.)

Example 1. Use factoring to solve the following equations.

(a) $6x^2 - 13x + 6 = 0.$

(c) $18x^3 + 3x^2 - 6x = 0$

(b) $4x^2 - 12x + 9 = 0$

(d) $2x^3 + 5x^2 - 8x - 20 = 0$

- **Square Root Method:** The solutions of the quadratic equation $x^2 = C$ are

$$x = \sqrt{C} \quad \text{and} \quad x = -\sqrt{C}.$$

These solutions can be combined as $x = \pm\sqrt{C}$. Note that this method can also be used to solve a quadratic equation of the form $(ax + b)^2 = c$.

Example 2. Use the square root method to solve the following quadratic equations.

(a) $3x^2 - 15 = 0$

(c) $2(3x + 1)^2 - 8 = 0$

(b) $5(x - 1)^2 - 40 = 0$

(d) $4(2x - 1)^2 - 20 = 0$

- **Completing the Square:** is a method used to covert one side of a quadratic equation into a perfect binomial square and use the square root method to solve the equation.

STEPS FOR COMPLETING THE SQUARE	EXAMPLE: $x^2 + 4x + 2 = 0$
1) Isolate the constant on one side of the equation.	$x^2 + 4x \ = -2$
2) Make sure the coefficient of x^2 is a positive one. If not, divide by this coefficient.	
3) Determine $\left(\frac{1}{2} \cdot \text{ coeff of } x\right)^2$	$\left(\frac{1}{2} \cdot 4\right)^2 = 2^2 = 4$
4) Add the result of step (3) to both sides.	$x^2 + 4x + 4 = -2 + 4$
5) Factor as a perfect square and solve by using the Square Root Property.	$(x+2)^2 = 2$ $\sqrt{(x+2)^2} = \sqrt{2}$ $x + 2 = \pm\sqrt{2}$ $x = -2 \pm \sqrt{2}$

Example 3. Complete the square to solve the following quadratic equations.

(a) $x^2 - 8x + 3 = 0$

(b) $3x^2 + 6x - 12 = 0$

- **Quadratic Formula:** The solutions of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
where a is the coefficient of x^2, b is the coefficient of x, and c is the constant term.

  * The quadratic formula is a result of solving $ax^2 + bx + c = 0$ by completing the square.
  * The quadratic formula can be used to solve *any* quadratic equation.
  * The expression $b^2 - 4ac$ is called the **discriminant**.
    - If $b^2 - 4ac < 0$ then there are two different complex number solutions to the quadratic equation.
    - If $b^2 - 4ac = 0$, then the quadratic equation has only one real zero.
    - If $b^2 - 4ac > 0$, then the quadratic equation has two different real solutions.

Example 4. Use the Quadratic Formula to solve the following quadratic equations.

(a) $2x^2 - 5x - 2 = 0$

(b) $20x^2 - 8x - 4 = 0$

Example 5. A tennis ball is thrown into a swimming pool from the top of a tall hotel. The height of the ball above the pool is modeled by $H(t) = -16t^2 - 4t + 200$ feet, where t is the time, in seconds, after the ball was thrown. How long after the ball is thrown is it 44 feet above the pool?

Example 6. The total revenue function for a product is given by $R = 266x$, and the total cost function for this same product is $C = 2000 + 46x + 2x^2$, where R and C are each measured in thousands of dollars and x is the number of units produced and sold.

(a) Form the profit function for this product from the two given functions.

(b) What is the profit when 55 units are produced?

(c) How many units must be sold to break even on this product? (i.e., give zero profit.)

Example 7. Solve the following equations:

(a) $(3x+2)^2 + 7(3x+2) - 8 = 0$

(b) $15(2x+1)^2 - 4(2x+1) - 3 = 0$