MATH 11010: CHAPTER #4 REVIEW

- **Section 4.1:**
  - Be able to find the inverse of a one-to-one function.
  - Be able to determine if a given function is a one-to-one function.

- **Section 4.2:**
  - Be able to recognize exponential functions and sketch their graphs.
  - Be able to use compound interest formula.

- **Section 4.3:**
  - Be able to evaluate a logarithmic expression. Recall for bases 10 and $e$ that
    \[ \log_{10} x = \log x \quad \text{and} \quad \log_e x = \ln x \]
  - Be able to switch from logarithmic form to exponential form and also from exponential form to logarithmic form:
    \[ y = \log_a x \quad \text{if and only if} \quad x = a^y. \]
  - Be able to find domain and vertical asymptote of a logarithmic function.
  - Be able to use the Change of Base formula.

- **Section 4.4:**
  - Be able to switch from single logarithm form to expanded form and from expanded form to a single logarithm.
  - Be able to use the Properties of Logarithms and the Laws of Logarithms.

- **Section 4.5:**
  - Be able to solve any exponential equation.
  - Be able to solve any logarithmic equation.

- **Section 4.6:**
  - Be able to solve any application problem involving compound interest, compounded continuously interest, exponential growth, and exponential decay. This means solving for time, growth/decay rate, principal/initial value, or amount after time $t$. 
PRACTICE PROBLEMS

NOTE: These problems are to be used for practice purposes only. DO NOT assume that your test will look exactly like these problems. Also, DO NOT assume that only these types of problems are on the exam.

1. Use the Change of Base Formula and a calculator to evaluate $\log_{25} 13$ correct to four decimal places.

2. Evaluate each logarithm.
   
   (a) $\log_4 \frac{1}{2} =$
   (b) $\log_{27} 9 =$
   (c) $\log_4 32 =$
   (d) $\log_5 \frac{1}{125} =$
   (e) $\log_{16} \frac{1}{8} =$
   (f) $\log_3 81 =$
   (g) $\log \left( \frac{1}{1000} \right) =$
   (h) $\log \sqrt{10} =$
   (i) $\ln e^8 =$
   (j) $\ln \left( \frac{1}{e^7} \right) =$
   (k) $\log_{1/5} 125 =$
   (l) $\log_{1/2} 32 =$

3. Write each equation in exponential form.
   
   (a) $\log_6 37 = y$
   (b) $\ln(x + 2) = 3$

4. Write each equation in logarithmic form.
   
   (a) $2^6 = 64$
   (b) $10^{-4} = \frac{1}{10000}$

5. Rewrite the following expression as a single logarithm.
   
   (a) $2 \ln x - 5 \ln(x + 3) - 4 \ln y$
   (b) $7 \ln x - 2 \left[ 3 \ln(z + 1) - 4 \ln y \right]$
   (c) $3 \ln(x - 2) - 5 \left[ \ln x - 2 \ln(x + 1) \right]$
   (d) $3 \log x + \frac{1}{2} \log y - \log 6 - 2 \log w$

6. Rewrite the following expression in expanded form. (Write as a sum, difference, or multiple of logarithms.)
   
   (a) $\ln \frac{3x}{2y}$
   (b) $\log \frac{4x^2}{y^3}$
   (c) $\ln \frac{5\sqrt{x}}{6y^2}$
   (d) $\log \frac{(x + 3)^2}{5x^2}$

7. Solve for $x$.
   
   (a) $\log_{36} x = \frac{3}{2}$
   (b) $\ln(7x + 3) = 0$
   (c) $2 \log(x + 3) = 4$
   (d) $\log_3 x + \log_3(2x + 5) = 1$
   (e) $\ln x + \ln(x - 2) = \ln 3$
   (f) $\log(x + 4) - \log(x - 5) = 2$
   (g) $\log(x + 2) + \log(x - 1) = 1$
   (h) $\ln 2 + \ln x = \ln(7 + x) - \ln(x - 2)$
   (i) $3 = \log_2(x - 2) + \log_2 7$
(j) \[5 + \log_3(2x + 1) = 8\]  
(k) \[\log_2(x + 6) - \log_2(x + 2) = \log_2 x\]

8. Solve for \(x\). Give both the exact answer and a decimal approximation, accurate to four decimal places.

(a) \[2^{2x-3} = 5\]  
(b) \[3(5 + e^{2x}) = 24\]  
(c) \[4^{5x-3} = 7\]  
(d) \[10^{2x+1} = 9\]

(e) \[600e^{0.027t} = 1800\]  
(f) \[200 \left( 1 + \frac{0.028}{4} \right)^{4t} = 800\]  
(g) \[7^{x-3} = 5^{x+2}\]  
(h) \[x^2e^x - 9e^x = 0\]

9. An investment of $4500 is compounded continuously at an annual percentage rate of 3.75%. How long does it take for the investment to double in value?

10. An investment of $400 is made at 2.25%, compounded continuously. How long will it take for the investment to double?

11. An investment grows at 4.23% compounded monthly. How many years will it take for the investment to increase by 75%?

12. The turtle population in local lake grows exponentially. The current population is 54 turtles and the relative growth rate is 21% per year. Find the number of years required for the turtle population to reach 500.

13. A family invests money in an account paying 3.25%, compounded monthly. At the end of 4 years the account has $12,345. Find the initial amount this family invested.

14. If $15,750 is invested in an account earning 5\(\frac{3}{4}\)% per year compounded continuously, determine the amount in the account at the end of 3 years.
15. Consider a rabbit population that grows exponentially at a rate of 8.2% per month. After 4 months, the population contains 68 rabbits.

(a) Find the initial number of rabbits in this population.

(b) Find the projected population after 18 months. (Round to nearest whole unit).

16. The half-life of radium-226 is 1590 years. Suppose we begin with a 150 mg sample.

(a) Find a function that models the mass remaining after $t$ years.

(b) Find the mass that will remain after 900 years? (Round to two decimal places)

(c) After how many years will only 50 mg of the sample remain? (Round to two decimal places)
ANSWERS

1. 0.7968

2. (a) $-\frac{1}{2}$
   (b) $\frac{2}{3}$
   (c) $\frac{5}{2}$
   (d) $-4$
   (e) $-\frac{3}{4}$
   (f) 4
   (g) $-3$
   (h) $\frac{1}{2}$
   (i) 8
   (j) $-7$
   (k) $-3$
   (l) $-5$

3. (a) $37 = 6^y$
   (b) $x + 2 = e^3$

4. (a) $\log_2 64 = 6$
   (b) $\log \frac{1}{10000} = -4$

5. (a) $\ln \frac{x^2}{(x + 3)^5y^4}$
   (b) $\ln \frac{x^7y^8}{(z + 1)^6}$
   (c) $\ln \frac{(x - 2)^3(x + 1)^{10}}{x^5}$
   (d) $\log \frac{x^3\sqrt{y}}{6w^2}$

6. (a) $\ln 3 + \ln x - \ln 2 - \ln y$
   (b) $\log 4 + 2 \log x - 3 \log y$
   (c) $\ln 5 + \frac{1}{2} \ln x - \ln 6 - 2 \ln y$
   (d) $2 \log (x + 3) - \log 5 - 2 \log x$

7. (a) $x = 216$
   (b) $x = -\frac{2}{7}$
   (c) $x = 97$
   (d) $x = \frac{1}{2}$; $x = -3$ does not check.
   (e) $x = 3$; $x = -1$ does not check.
   (f) $x = \frac{504}{99}$
   (g) $x = 3$; $x = -4$ does not check.
   (h) $x = \frac{7}{2}$; $x = -1$ does not check.
(i) \( x = \frac{22}{7} \)  
(j) \( x = 13 \)  
(k) \( x = 2; (x = -3 \text{ does not check.}) \)  
(l) \( x = \frac{5}{2}; (x = -2 \text{ does not check.}) \)

8. (a) \( x = \frac{\ln 5}{2 \ln 2} + \frac{3}{2} \approx 2.6610 \)  
(b) \( x = \frac{\ln 3}{2} \approx 0.5493 \)  
(c) \( x = \frac{3 \ln 4 + \ln 7}{5 \ln 4} \approx 0.8807 \)  
(d) \( x = \frac{-1 + \log 9}{2} \approx -0.0229; \) NOTE: answer can also be written as \( \frac{\ln 9 - \ln 10}{2 \ln 10} \)  
(e) \( t = \frac{\ln 3}{0.027} \approx 40.6893 \)  
(f) \( t = \frac{\ln 4}{4 \ln(1.007)} \approx 49.6836 \)  
(g) \( x = \frac{2 \ln 5 + 3 \ln 7}{\ln 7 - \ln 5} \approx 26.9164 \)  
(h) \( x = \pm 3 \)

9. 18.48 years  
10. 30.81 years  
11. 13.25 years  
12. 10.60 years  
13. \( P = $10,841.99 \)  
14. \( A = $18,715.28 \)  
15. (a) 49 rabbits  
(b) 214 rabbits  
16. (a) \( M = 150e^{-\frac{\ln 2}{1590}t} \)  
(b) \( M = 101.32 \text{ mg} \)  
(c) \( t = 2520.09 \text{ years} \)