MATH 11010:   Exam #3 (Fall 2011)

1. Suppose a polynomial function with rational coefficients has

$$8i, \qquad 3 - 5\sqrt{7}, \qquad \frac{2}{3}, \qquad \text{and} \qquad 5 - 3i$$

   as some of its zeros. List the values that must also be zeros.

2. Find a polynomial function with zeros $x = 2$ (multiplicity 3) , $x = -9$ (multiplicity 1), and $x = 0$ (multiplicity 2). You may leave your answer in factored form.

3. Use the Rational Zero Theorem to list all possible rational zeros of

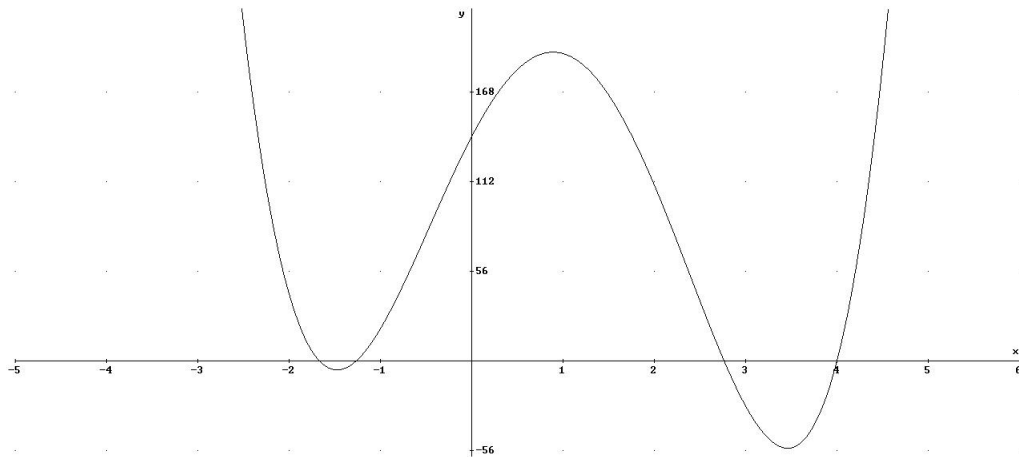$$f(x) = 4x^6 - 3x^4 + 2x - 9.$$

4. Use synthetic division to divide

$$P(x) = 3x^5 - 4x^3 + 9x^2 + 2x - 5 \qquad \text{by} \qquad x + 2.$$

   Identify the quotient and remainder. Be specific.

5. Use synthetic division to find $P\left(-\dfrac{3}{4}\right)$ for $P(x) = 8x^4 + 2x^3 + 13x^2 - 4x + 5$. Be sure to label your answer.

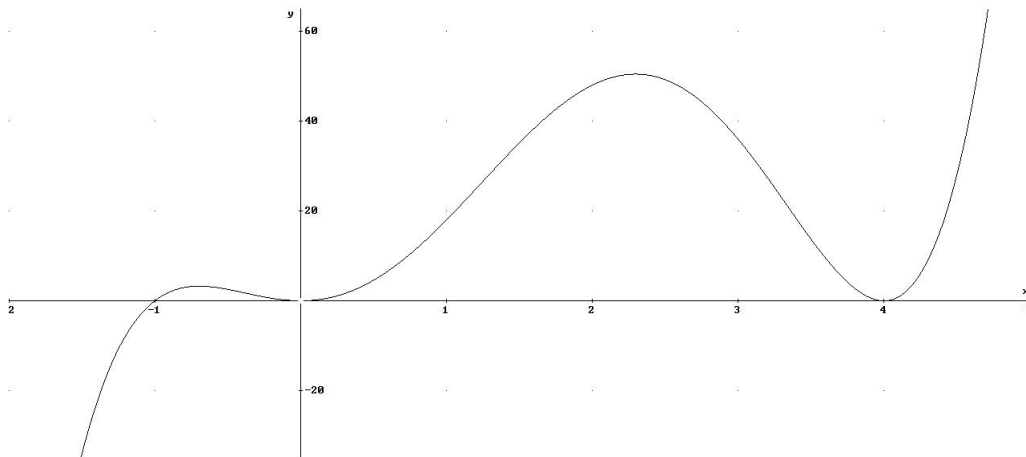6. Let $P(x) = 6x^4 - 23x^3 - 40x^2 + 109x + 140$.

   (a) Show that $x = -\dfrac{5}{3}$ is a zero.

   (b) Find all other zeros of P. You must show all work. (Show algebraically that they are zeros).

7. Consider $P(x) = -18x^4 + 63x^3 - 34x^2 - 44x + 8$ whose graph is given below. Find all zeros of this function. (Show algebraically that they are zeros).
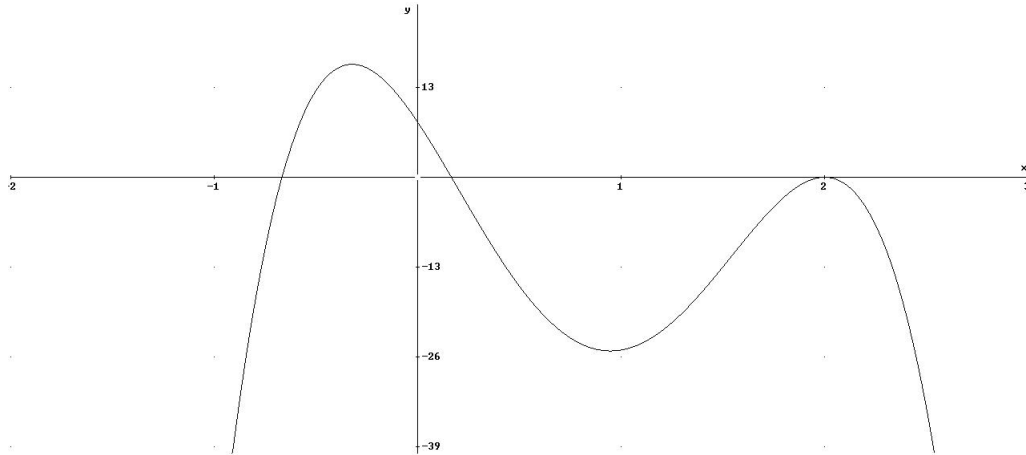
8. Given below is the graph of f.

   (a) Is the degree of f even or odd?

   (b) Is the leading coefficient of f positive or negative?

   (c) Determine the interval(s) where $f(x) > 0$.

   (d) Determine the interval(s) where $f(x) < 0$.

   (e) List the real zeros of f AND state whether each zero has even or odd multiplicity.

9. Find the equation of the vertical asymptote(s) of the following functions. Be specific!
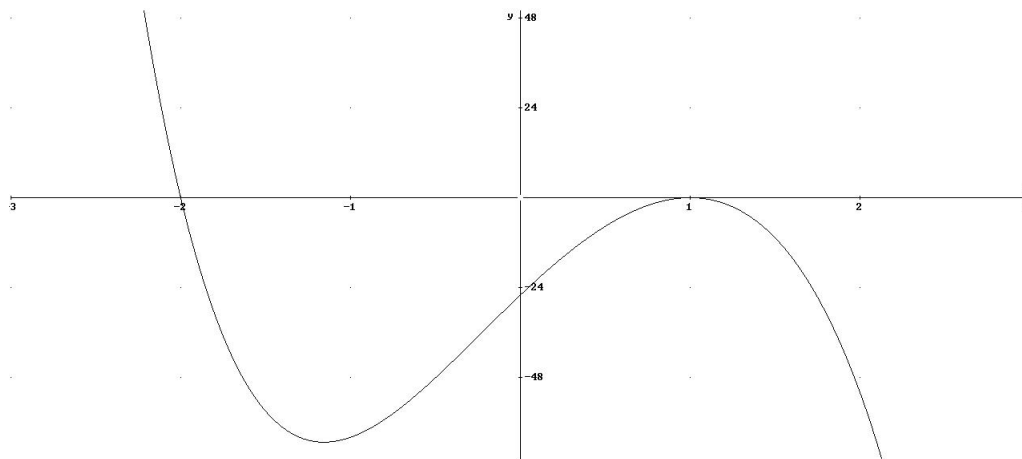
(a) $f(x) = \dfrac{6x^2 - 5x + 4}{21x^2 - x - 10}$

(b) $g(x) = \dfrac{3x + 11}{27x^3 - 15x}$

10. Find the equation of the horizontal asymptote of the following functions, if one exists. Be specific!

(a) $f(x) = \dfrac{7x^4 - 11x + 9}{3x^2 - x - 2}$

(b) $g(x) = \dfrac{8x^2 - 2x - 3}{25x^2 - 16}$

11. Find all zeros of $P(x) = -x^5 + 2x^4 - 10x^3 - 8x^2 + 43x - 26$.

12. Find a rational function that satisfies the following conditions:

$$\text{Vertical Asymptotes:} \quad x = -\frac{2}{3}, \quad x = \frac{1}{4}$$
$$\text{Horizontal Asymptote:} \quad y = \frac{5}{6}$$

13. Solve: $6x^3 + 8x^2 > 0$

14. Solve: $8x^2 - 18x - 5 \geq 0$

15. Solve: $\dfrac{x - 6}{4x^2 - 4x - 3} \leq 0$

ANSWERS

1. $-8i,\ \ 3 + 5\sqrt{7},\ \ 5 + 3i$

2. $f(x) = x^2(x-2)^3(x+9)$

3. $\pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}, \pm \frac{9}{4}$

4. See HW #4

5. $P\left(-\frac{3}{4}\right) = 17$

6. (a) Show using synthetic division that the remainder is zero.   (b) $x = 4,\ \ x = \dfrac{3 \pm \sqrt{65}}{4}$

7. $x = 2$ (multiplicity 2), $\ \ x = \dfrac{1}{6},\ \ x = -\dfrac{2}{3}$

8. (a) odd

   (b) positive

   (c) $(-1, 0) \cup (0, 4) \cup (4, \infty)$

   (d) $(-\infty, -1)$

   (e) $x = -1$ odd, $\ \ x = 0$ even, $\ \ x = 4$ even

9. (a) $x = \dfrac{5}{7},\ \ x = -\dfrac{2}{3}$ (b) $x = 0,\ \ x = \pm \dfrac{\sqrt{5}}{3}$

10. (a) no horizontal asymptote   (b) $y = \dfrac{8}{25}$

11. See HW #4

12. See HW #5

13. $\left(-\dfrac{4}{3}, 0\right) \cup (0, \infty)$ 14. See HW #5   15. See HW #5