MATH 11010: Exam #3 (Spring 2013)

1. Solve: $6x^2 - x - 15 \geq 0$

2. Solve: $\dfrac{3x-2}{8x^2+14x+3} \leq 0$

3. Find the equation of the vertical asymptote(s) of the following functions. Be specific!

   (a) $f(x) = \dfrac{x^2-3x+2}{49x^2-8}$ (b) $g(x) = \dfrac{4x^2-1}{5x^3-2x^2-20x+8}$

4. Find the equation of the horizontal asymptote of the following functions, if one exists. Be specific!

   (a) $f(x) = \dfrac{x^2-3x+2}{49x^2-8}$ (b) $g(x) = \dfrac{4x^2-1}{5x^3-2x^2-20x+8}$

5. Consider $f(x) = \dfrac{4x^3-6x^2+x-15}{2x^2+x+3}$

   (a) Explain how you know that f has an oblique asymptote. Be specific.

   (b) Find the equation of the oblique asymptote.

6. Find a rational function that satisfies the following conditions:

$$\text{Vertical Asymptotes:} \quad x = 3, \quad x = -\frac{5}{3}$$

$$\text{Horizontal Asymptote:} \quad y = \frac{5}{12}$$

7. Use the Rational Zero Theorem to list all possible rational zeros of

$$f(x) = 9x^5 - 6x^4 + 3x^3 - 7x + 8.$$

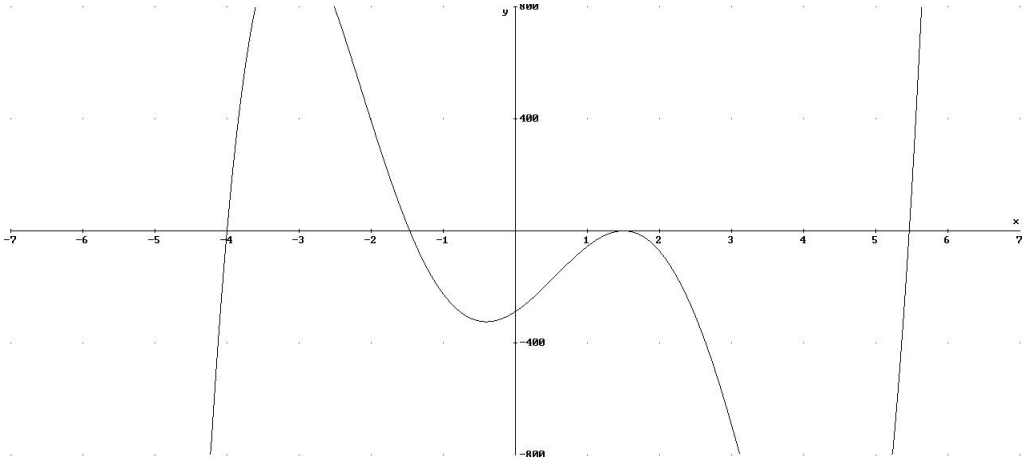
8. Suppose a polynomial function with rational coefficients has

$$7 - 5i, \qquad \sqrt{5}, \qquad \frac{7}{8}, \qquad \text{and} \qquad -3$$

as some of its zeros. List the values that must also be zeros.

9. Find a polynomial function with zeros $x = \frac{7}{9}$ (multiplicity 3) , $x = 0$ (multiplicity 2), and $x = -3$ (multiplicity 4). Please leave your answer in factored form.

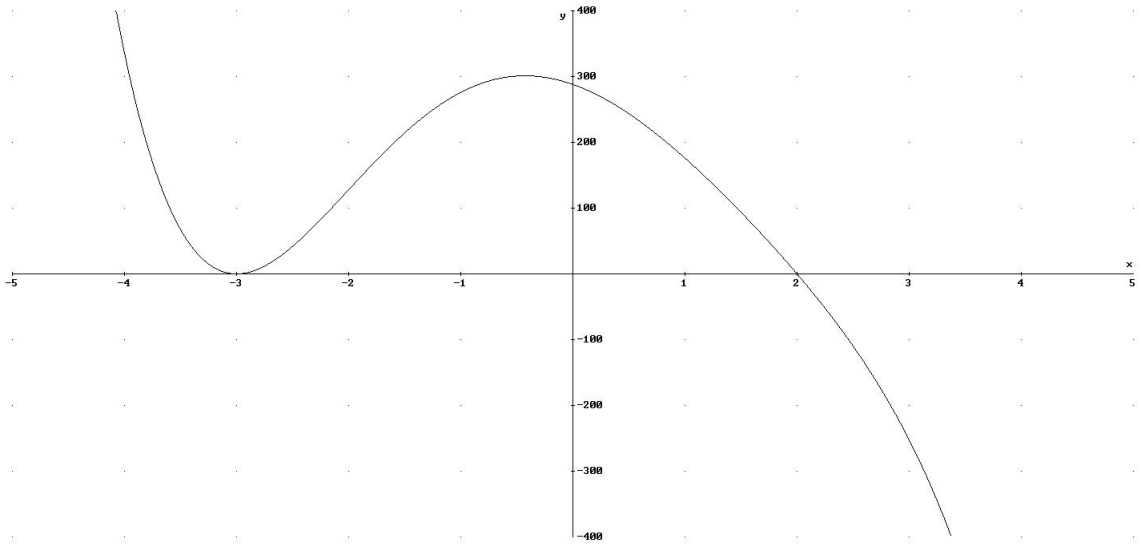
10. Consider $P(x) = 4x^5 - 12x^4 - 87x^3 + 160x^2 + 168x - 288$ whose graph is given below.



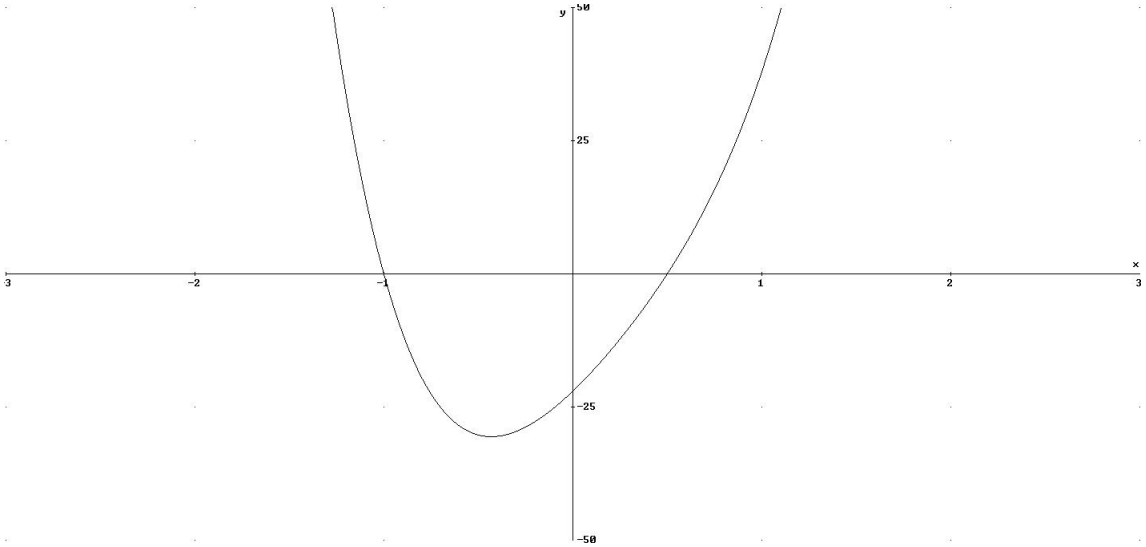
(a) Show that $x = \frac{3}{2}$ is a zero.

(b) Find all other zeros of P. You must show all work. (Show algebraically that they are zeros).

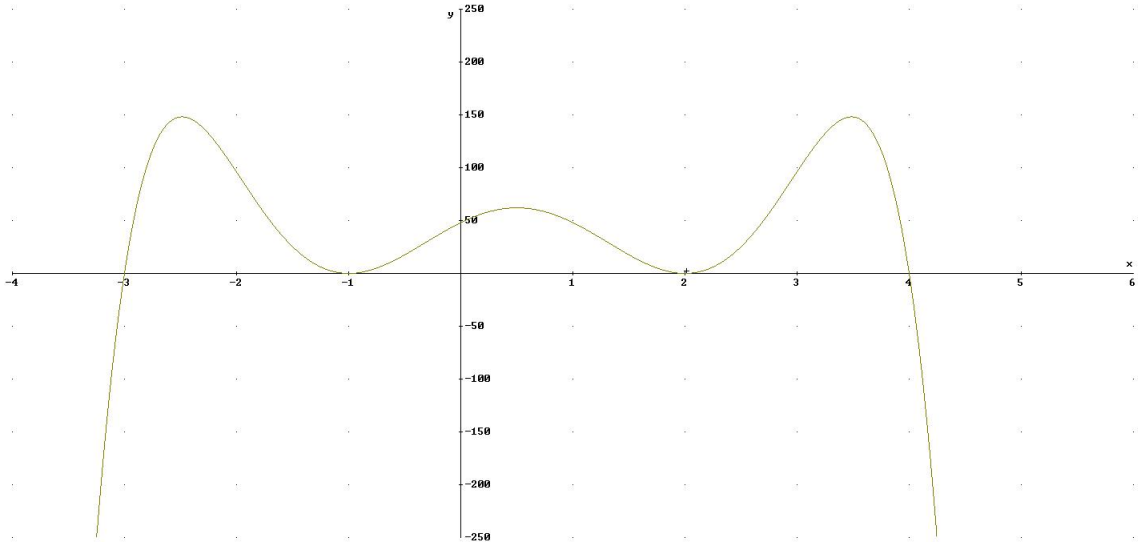
11. Consider $P(x) = -x^5 + 2x^4 + 11x^3 - 64x^2 - 60x + 288$ whose graph is given below. Find all zeros of this function. (Show algebraically that they are zeros).



12. Find all zeros of $P(x) = 18x^4 - 15x^3 + 23x^2 + 34x - 22$.



13. Given below is the graph of f.



(a) Is the degree of f even or odd?

(b) Is the leading coefficient of f positive or negative?

(c) Determine the interval(s) where $f(x) \geq 0$.

(d) Determine the interval(s) where $f(x) < 0$.

(e) List the real zeros of f AND state whether each zero has even or odd multiplicity.

ANSWERS

1. $\left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{5}{3}, \infty\right)$

2. $\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{1}{4}, \frac{2}{3}\right]$

3. (a) $x = \pm\frac{2\sqrt{2}}{7}$ (b) $x = \frac{2}{5},\ x = \pm 2$

4. (a) $y = \frac{1}{49}$ (b) $y = 0$

5. (a) the degree of the numerator is exactly one more than the degree of the denominator.
   (b) $y = 2x - 4$

6. $f(x) = \dfrac{5x^2}{12x^2 - 16x - 60}$

7. $\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{1}{9}, \pm\frac{2}{9}, \pm\frac{4}{9}, \pm\frac{8}{9}$

8. $7 + 5i, \quad -\sqrt{5}$

9. $f(x) = x^2(9x - 7)^3(x + 3)^4$

10. (a) show using synthetic division that the remainder is zero.
    (b) $x = \frac{3}{2},\ x = -4,\ x = 2 \pm 2\sqrt{3}$

11. $x = -3$ (multiplicity 2), $x = 2,\ x = 3 \pm \sqrt{7}\, i$

12. $x = \frac{1}{2},\ x = -1,\ x = \frac{2}{3} \pm \sqrt{2}\, i$

13. (a) even
    (b) negative
    (c) $[-3, 4]$
    (d) $(-\infty, -3) \cup (4, \infty)$
    (e) $x = -3$ (odd); $x = -1$ (even); $x = 2$ (even); $x = 4$ (odd)