MATH 11010: More Equations Section 2.5

- **Rational Equations:** A **rational equation** is an equation containing rational expressions. One method to solve a rational equation is to multiply both sides by the least common denominator to clear the equation of fractions.

  * CAUTION: When each side of a rational equation is multiplied by a variable expression, the resulting answers may not satisfy the original equation. Check all answers in the original equation and exclude any that make the denominator zero.

Example 1: Solve the following.

(a) $\dfrac{2x}{x+7} - \dfrac{5}{x+1} = 0$

(b) $\dfrac{6x}{3x+1} = \dfrac{1}{x} - \dfrac{2}{3x+1}$

- **Radical equations:** A **radical equation** is an equation in which variables appear in one or more radicands. To solve a radical equation we can use the following prinicple:

  For any positive integer n, if $a = b$ is true, then $a^n = b^n$ is true.

- CAUTION: The converse of the above statement is not true. For example, $(-4)^2 = (4)^2$ is true, but $-4 \neq 4$.

Example 2: Solve the following equations.

(a) $\sqrt{x+3} - 1 = x$

(b) $x - 1 = \sqrt{4x-7}$

(c) $3x^2\sqrt{x+5} = 0$

- **Absolute Value Equations:** Let $c > 0$. Then

$$|ax+b| = c \qquad \text{is equivalent to} \qquad ax+b=c \quad \text{or} \quad ax+b=-c$$

* CAUTION: Before rewriting the absolute value equation, make sure the absolute value is isolated on one side. Do NOT rewrite until the absolute value is isolated.

Example 3: Solve the following equations.

(a) $|3x-5|+2=9$

(b) $11-|4-5x|=3$

- **u-substitution Equations:** When an expression contains a quantity in parentheses, sometimes it is beneficial to use a u-substitution.

Example 4: Solve the following equations.

(a) $10x(x-2)^3 + 15x^2(x-2)^2 = 0$

(b) $(3x+1)^2 - 5(3x+1) - 24 = 0$

(c) $6(2x-3)^2(4x+5)^2 + 8(2x-3)^3(4x+5) = 0$

- **Higher Order Equations:** Set the equation equal to zero, then factor and split. Be sure to keep in mind some of the advanced factoring techniques (e.g., grouping and quadratic type), as well as the four techniques to solve a quadratic equation.

Example 5: Solve the following equations.

(a) $4x^3 + 6x^2 - 40x = 0$

(b) $18x^3 + 9x^2 - 16x - 8 = 0$

(c) $9x^4 - 17x^2 + 8 = 0$

- **Rational Exponent Equations:**

Example 6: Solve the following equations.

(a) $x^{3/2} - 8 = 0$

(b) $x^{2/3} - 4 = 0$

(c) $x^{\frac{4}{3}} + 2x^{\frac{2}{3}} - 3 = 0$

(d) $2\sqrt{6x+5} + \dfrac{3(2x+1)}{\sqrt{6x+5}} = 0$