MATH 11010: Algebra of Functions
Section 1.6

Sums, Differences, Products, and Quotients

If f and g are functions and x is in the domain of each function, then

- $(f+g)(x) = f(x) + g(x)$
- $(f-g)(x) = f(x) - g(x)$
- $(fg)(x) = f(x) \cdot g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$

NOTES:

- The domain of $f+g$, $f-g$, and fg is the intersection of the domain of f and the domain of g.
- The domain of $\frac{f}{g}$ is also the intersection of the domain of f and the domain of g with the exclusion of any values of x for which $g(x) = 0$.

Example 1: Given $f(x) = 3x - 4$ and $g(x) = \sqrt{2x-1}$, find each of the following, if it exists.

(a) $(f+g)(5)$

(b) $\left(\frac{f}{g}\right)(4)$

(c) $(fg)(0)$

(d) domain of $\frac{g}{f}$

Difference Quotients: $\dfrac{f(x+h)-f(x)}{h}$

Example 2: Given $f(x) = 2x^2 - 5x + 7$, find $\dfrac{f(x+h)-f(x)}{h}$.

Example 3: Given $f(x) = \dfrac{x}{2-x}$, find $\dfrac{f(x+h)-f(x)}{h}$.

Composition of Functions

The **composition function** $f \circ g$ is defined as

$$(f \circ g)(x) = f(g(x)),$$

where x is in the domain of g and $g(x)$ is in the domain of f.

Example 4: Let $f(x) = 3x - 2$ and $g(x) = 5 - 3x - 2x^2$. Find and simplify each of the following.

(a) $(f \circ g)(x) =$

(b) $(g \circ f)(x) =$

(c) $(f \circ f)(-2) =$

(d) $(g \circ g)(0) =$

Example 5: Let $H(x) = 3(2x+1)^5 - 7$. Find functions f and g such that

$$(f \circ g)(x) = H(x)$$

Homework: pp 143–145; #1–31 eoo, 39, 41, 47–101 eoo