MATH 11010:   Exp and Log Equations Section 4.5

- **Exponential functions:** The function $f(x) = a^x$, where x is a real number, $a > 0$ and $a \neq 1$, is called an **exponential function** with base a.

- **Guidelines for solving exponential equations**:

  1. Isolate the exponential expression on one side of the equation. (If the exponential expression cannot be isolated, try factoring the equation. Then set each factor equal to zero and continue to step 2.)

  2. Take the logarithm of each side in order to "bring down" the variable in the exponent. (It does not matter which base you use on the logarithm as long as it is the same on both sides.)

  3. Solve for the variable.

- **Base-Exponent Property:** For any $a > 0$, $a \neq 1$,

$$\text{if} \quad a^x = a^y \quad \text{then} \quad x = y$$

Example 1: Solve: $5^x = 20$

Example 2: Solve: $3^{x-5} = 11$

Example 3: Solve: $4\left(1 + e^{3x}\right) = 20$

Example 4: Solve: $e^x - 6e^{-x} = 1$

- **Logarithmic functions:** Let a be a positive number with $a \neq 1$. The **logarithmic function** with base a, denoted $\log_a x$, is defined by

$$\boxed{y = \log_a x \qquad \text{if and only if} \qquad x = a^y.}$$

- **Guidelines for solving logarithmic equations:**

  1. Isolate the logarithmic term on one side of the equation. This is accomplished by using the laws of logarithms.

  2. Write the equation in exponential form.

  3. Solve for the variable.

  4. Check to make sure you don't have extraneous solutions. To do this, substitute "answers" into the original equation and check that you are not taking the logarithm of a negative number or zero.

- **Property of Logarithmic Equality:** For any $M > 0$, $N > 0$, $a > 0, a \neq$,

$$\text{if} \quad \log_a M = \log_a N \quad \text{then} \quad M = N$$

Example 5: Solve: $\quad \log_2(x - 1) = 3$

Example 6: Solve: $\quad \log_2(x + 1) + \log_2(x - 1) = 3$

Example 7: Solve: $\ln(x - 1) + \ln(x + 3) = \ln 5$

Example 8: Solve: $\log 3x - \log(x - 2) = 1$

Homework: pp 408;   1–51 odd, plus additional exercises 1-21 odd

ADDITIONAL EXERCISES

Solve for x.

1. $\log(7x + 2) = 1$

2. $2\log(x + 3) = 4$

3. $3\log(2x + 1) - 2 = 1$

4. $\log x - \log(3x - 2) = 2$

5. $5\ln(2x - 3) + 2 = 2$

6. $\ln(2x - 3) - \ln(x - 1) = 0$

7. $\ln(3x - 2) = \ln(2x + 5)$

8. $2\ln x - \ln(x + 12) = 0$

9. $\log x + \log(x - 3) = 1$

10. $\ln(8x - 3) = \ln(4x + 1)$

11. $\log(x - 1) + \log(x - 3) = 0$

12. $\ln x + \ln(x + 3) = \ln(x + 8)$

13. $\log(x + 3) + \log(x + 1) = \log(x + 7)$

14. $\log 3x - \log(x - 2) = 1$

15. $\ln(x^2 - 12) - \ln(3x - 2) = 0$

16. $\ln(x^2 - 8) - \ln(x + 4) = 0$

17. $x^2 \ln x - 5\ln x = 0$

18. $(x - 1)\ln(x + 5) = 0$

19. $2x\ln x - 8\ln x = 0$

20. $3e^x \ln(4x - 3) = 0$

21. $(x^2 - 9)\ln(x - 1) = 0$

22. $3(x - 7)\ln(e^x - 1) = 0$

ANSWERS

1. $x = 8/7$

2. $x = 97$

3. $x = 9/2$

4. $x = 200/299$

5. $x = 2$

6. $x = 2$

7. $x = 7$

8. $x = 4$

9. $x = 5$

10. $x = 1$

11. $x = 2 + \sqrt{2}$

12. $x = 2$

13. $x = 1$

14. $x = 20/7$

15. $x = 5$

16. $x = 4, \ -3$

17. $x = 1, \ \sqrt{5}$

18. $x = 1, \ -4$

19. $x = 1, \ 4$

20. $x = 1$

21. $x = 2, \ 3$

22. $x = \ln 2, \ 7$