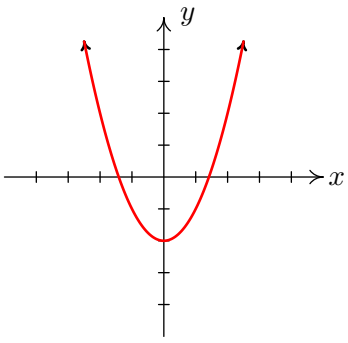
MATH 11010: Inverse Functions
Section 4.1

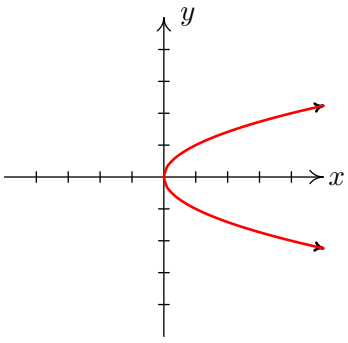
- **Inverse Relation:** Interchanging the first and second coordinates of each ordered pair in a relation produces the **inverse relation**. If the relation is defined by an equation, interchanging the variables produces an equation of the inverse relation.
- **One-to-one functions:** A function f is **one-to-one** if no two elements of the domain A have the same image. In other words, f is a one-to-one function if $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

Example 1: Determine whether $f(x) = 8x - 3$ is a one-to-one function.

- **Horizontal Line Test:** If it is possible for a horizontal line to intersect the graph of a function more than once, then the function is NOT one-to-one.

Example 2: Determine if each of following curves is the graph of an one-to-one function.





- **Inverse function**: Let f be a one-to-one function with domain A and range B. Then its **inverse function**, denoted f^{-1}, has domain B and range A and is defined by

$$f^{-1}(y) = x \qquad \text{if and only if} \qquad f(x) = y$$

for any y in B. Please note that -1 is NOT an exponent; therefore, f^{-1} does NOT mean the reciprocal of f.

- **Properties of inverse functions**:

  * Let f be a one-to-one function with domain A and range B. The inverse function f^{-1} satisfies

$$\begin{aligned} & f^{-1}(f(x)) = x \qquad \text{for every } x \text{ in } A \\ \text{and} \qquad & f(f^{-1}(x)) = x \qquad \text{for every } x \text{ in } B \end{aligned}$$

  * The inverse of f^{-1} is f. So, we say that f and f^{-1} are inverses of each other.
  * The inverse function interchanges the domain and range. Namely,

$$\begin{aligned} \text{Domain of } f &= \text{Range of } f^{-1} \\ \text{Range of } f &= \text{Domain of } f^{-1} \end{aligned}$$

  * The graph of f^{-1} is found by reflecting the graph of f across the line $y = x$.
  * Only a one-to-one function can have an inverse function.

Example 3: For the function f, use composition of functions to show that f^{-1} is as given.

$$f(x) = \frac{x+5}{4} \qquad \text{and} \qquad f^{-1}(x) = 4x - 5.$$

Finding the inverse of a one-to-one function:

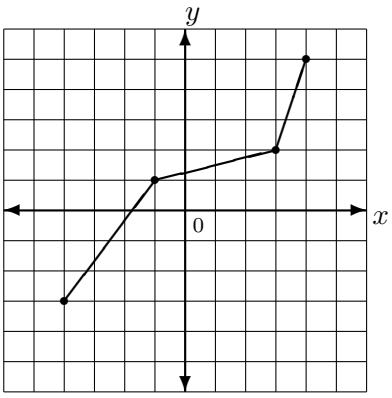
- Replace $f(x)$ with y.
- Interchange x and y.
- Solve this equation for y. The resulting equation is $f^{-1}(x)$.

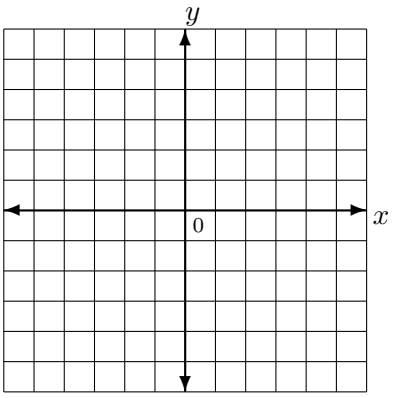
Example 4: Find the inverse of $f(x) = 9 - 7x$.

Example 5: Find the inverse of $f(x) = \dfrac{x+1}{3x+2}$

Example 6: Find the inverse of $f(x) = 3x^2 - 4, \quad x \leq 0.$

Example 7: Given the graph of f, sketch the graph of f^{-1}.





Homework: pp 356–358; #17–53 odd, 57, 59, 67–77 odd, 79–85 (just find inverse).