MATH 11010: Properties of Log Functions Section 4.4

- **Logarithmic functions:** Let a be a positive number with $a \neq 1$. The **logarithmic function** with base a, denoted $\log_a x$, is defined by

$$\boxed{y = \log_a x \qquad \text{if and only if} \qquad x = a^y.}$$

- **Common logarithm**: The logarithm with base 10 is called the common logarithm. The base 10 is usually omitted when working with the common logarithm.

$$\log_{10} x = \log x.$$

- **Natural logarithm**: The logarithm with base e is called the natural logarithm and is denoted by

$$\log_e x = \ln x.$$

Laws of Logarithms: Let a be a positive number with $a \neq 1$. Let $A > 0, \quad B > 0$, and n be any real number.

- $\log_a AB = \log_a A + \log_a B.$ (The logarithm of a product is the sum of the logarithms.)
- $\log_a \left(\frac{A}{B}\right) = \log_a A - \log_a B.$ (The logarithm of a quotient is the difference of the logarithms.)
- $\log_a A^n = n \log_a A.$ (The logarithm of a quantity raised to a power is the same as the power times the logarithm of the quantity.)

Common Mistakes to Avoid:

- $\log_a(A + B) \neq \log_a A + \log_a B.$
- $\log_a \frac{A}{B} \neq \frac{\log_a A}{\log_a B}.$
- $(\log_a A)^n \neq n \log_a A.$
- $\log_a AB \neq (\log_a A)(\log_a B).$

Example 1: Express as a single logarithm:

(a) $5\log z - 3\log x + 7\log y$

(b) $3\ln(x-2) - 5\left[\ln x - 2\ln(x+1)\right]$

(c) $4\left[3\log_2 x - \log_2(x+5)\right] - 2\log_2(x-5)$

Example 2: Express in expanded form:

(a) $\log_2 x^3y^5z^9$

(b) $\log\left(\dfrac{x^2(x+1)^6}{(x-3)^5}\right)$

(c) $\log_3\left(\dfrac{x^3}{\sqrt{x+1}(x-9)^7}\right)$

- **Properties of logarithms**: Let a be a positive number such that $a \neq 1$. Then
  - $\log_a 1 = 0$
  - $\log_a a = 1$
  - $\log_a a^x = x$
  - $a^{\log_a x} = x$

Example 3: Given that $\log_a 2 \approx 0.301$, $\log_a 7 \approx 0.845$, $\log_a 11 \approx 1.041$, find each of the following, if possible. Round answer to the nearest thousandths.

(a) $\log_a \frac{14}{11}$

(b) $\log_a 98$

Example 4: Simplify.

(a) $\log_9 9^{311} =$

(c) $e^{\ln 4t} =$

(b) $\ln e^{45} =$

(d) $10^{\log 78} =$

Homework: pp 396–397; 1–75 odd