
MATH 11010: Logarithmic Functions

Section 4.3

- **Logarithmic functions:** Let a be a positive number with $a \neq 1$. The **logarithmic function** with base a , denoted $\log_a x$, is defined by

$$\boxed{y = \log_a x \quad \text{if and only if} \quad x = a^y.}$$

- **Common logarithm:** The logarithm with base 10 is called the common logarithm. The base 10 is usually omitted when working with the common logarithm.

$$\log_{10} x = \log x.$$

- **Natural logarithm:** The logarithm with base e is called the natural logarithm and is denoted by

$$\log_e x = \ln x.$$

- **Properties of the graph of $y = \log_a x$:**

- * Domain is $(0, \infty)$.
- * Range is all real numbers.
- * Always crosses through the point $(1, 0)$.
- * $x = 0$ is a vertical asymptote.
- * The function is one-to-one.
- * If $a > 1$, then the function is increasing; if $0 < a < 1$, then the function is decreasing.

- **Properties of logarithms:** Let a be a positive number such that $a \neq 1$. Then

$$* \log_a 1 = 0$$

$$* \log_a a^x = x$$

$$* \log_a a = 1$$

$$* a^{\log_a x} = x$$

Example 1: Find each of the following:

(a) $\log_2 64 =$

(c) $\log_{1/27} 9 =$

(b) $\log_8 4 =$

(d) $\log_5 5^2 =$

Example 2: Convert to a logarithmic equation.

(a) $3^4 = 81$

(b) $10^3 = 1000$

(c) $e^4 = x$

Example 3: Convert to an exponential equation.

(a) $\log 7 = 0.845$

(b) $\ln 0.38 = -0.9676$

(c) $y = \log_4 7$

- **Change of base formula:** For any logarithmic bases a and b , and any positive number M ,

$$\log_b M = \frac{\log_a M}{\log_a b}.$$

The change of base formula allows you to use your calculator to evaluate logarithms. In order to use the calculator, a must be either 10 or e .

Example 4: Find $\log_7 9$ using a calculator. Round answer to four decimal places.

Example 5: Find the domain and vertical asymptote for $f(x) = \ln(4x - 7)$

Homework: pp 387–388; 1– 77 eoo, 83– 89 odd (Find domain and vertical asymptote only.)