MATH 11010: Polynomial Division
Section 3.3

- **Polynomial:** A **polynomial function** P is given by

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0$$

where the coefficients $a_0, a_1, \ldots, a_{n-1}, a_n$ are real numbers and the exponents are whole numbers.

Example 1: In the following, a polynomial P and a divisor d are given. Use long division to find the quotient Q and the remainder R when P is divided by d. Express P in the form $P(x) = d(x) \cdot Q(x) + R(x)$.

$$P(x) = x^3 - x^2 - 2x + 6 \qquad \text{and} \qquad d(x) = x - 2$$

- **Synthetic division**: is a quick method of dividing polynomials when the divisor is of the form $x - c$ where c is any constant (positive or negative).

Steps to divide $P(x)$ by $x - c$ using synthetic division: Synthetic division will consist of three rows.

1. Write the c and the coefficients of the dividend in descending order in the first row. If any x terms are missing, place a zero in its place.
2. Bring the leading coefficient in the top row down to the bottom (third) row.
3. Next, multiply the first number in the bottom row by c and place this product in the second row under the next coefficient and add these two terms together.
4. Continue this process until you reach the last column.
5. The numbers in the bottom row are the coefficients of the quotient and the remainder. The quotient will have one degree less than the dividend.

Common Mistakes to Avoid:

* Do NOT forget to record a zero for any missing terms. For example, suppose the dividend is $f(x) = 3x^4 - 5x^2 - 2$. Since both the x^3 and x terms are missing we would record the coefficients as $3 \;\; 0 \;\; -5 \;\; 0 \;\; -2$.
* Remember to add the terms inside the synthetic division process.
* If the divisor is $x + c$, then the number outside the synthetic division is $-c$. For example, if the divisor is $x + 5$ then we record a -5 on the outside of the synthetic division.

Example 2: Use synthetic division to find the quotient and remainder.

$$(x^3 - 3x + 10) \div (x - 2)$$

Example 3: Use synthetic division to find the quotient and remainder.

$$(4x^4 - 2x + 5) \div (x + 3)$$

- **Remainder Theorem**: If a polynomial P is divided by $x - c$, then the remainder is $P(c)$. This gives us another way to evaluate a polynomial at c.
- **Factor Theorem**: c is a zero of $P(x)$ if and only if $x - c$ is a factor of $P(x)$.

Example 4: Use synthetic division to find the function values:

(a) $f(x) = 2x^4 + x^2 - 10x + 1$; Find $f(2)$

(b) $P(x) = 4x^4 + 5x^2 - 9x + 7$; Find $P(-\frac{1}{2})$

Example 5: Determine whether $\frac{1}{3}$ and -2 are zeros of $f(x) = x^3 - x^2 - \frac{1}{9}x + \frac{1}{9}$.