MATH 11010: Polynomial Functions Section 3.1

- **Polynomial:** A **polynomial function** P is given by

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdot + a_1 x + a_0$$

where the coefficients $a_0, a_1, \ldots, a_{n-1}, a_n$ are real numbers and the exponents are whole numbers.

  * The coefficient a_n is called the **leading coefficient**.
  * The term $a_n x^n$ is called the **leading term**.
  * The **degree** of the polynomial is n.

- **Leading Term Test:** If $a_n x^n$ is the leading term of a polynomial function, then the left end behavior (as $x \to -\infty$) and right end behavior (as $x \to \infty$) of the graph can be described in one of the following ways:

  * If n is even and $a_n > 0$, then

$$f(x) \to \infty \text{ as } x \to -\infty \qquad \text{and} \qquad f(x) \to \infty \text{ as } x \to \infty$$

  * If n is even and $a_n < 0$, then

$$f(x) \to -\infty \text{ as } x \to -\infty \qquad \text{and} \qquad f(x) \to -\infty \text{ as } x \to \infty$$

  * If n is odd and $a_n > 0$, then

$$f(x) \to -\infty \text{ as } x \to -\infty \qquad \text{and} \qquad f(x) \to \infty \text{ as } x \to \infty$$

  * If n is odd and $a_n < 0$, then

$$f(x) \to \infty \text{ as } x \to -\infty \qquad \text{and} \qquad f(x) \to -\infty \text{ as } x \to \infty$$

- **Zero**: If P is a polynomial and if c is a number such that $P(c) = 0$ then c is a zero of P.

- The following are all equivalent:

  * c is a zero of P
  * $(c, 0)$ is an $x-$intercept of the graph of P
  * $x - c$ is a factor of P
  * $x = c$ is a solution of the equation $P(x) = 0$

- **Even and Odd Multiplicity:** Let $k \geq 1$. If $(x - c)^k$ is a factor of a polynomial function P and $(x - c)^{k+1}$ is not a factor of P and:

  * k is odd, then the graph crosses the $x-$axis at $(c, 0)$.
  * k is even, then the graph is tangent to the $x-$axis at $(c, 0)$.

- Every polynomial function of degree n, with $n \geq 1$ has at least one zero and at most n zeros.