MATH 11010: Zeros of Polynomials Section 3.4

- **Fundamental Theorem of Algebra:** Every polynomial function of degree n, with $n \geq 1$, has at least one complex zero. (Remember a real number is a complex number with imaginary part zero.)
- Every polynomial function f of degree n, with $n \geq 1$, can be factored into n linear factors (not necessarily unique).

Example 1: Find a polynomial of degree 4 with -2 as a zero of multiplicity 1, 3 as a zero of multiplicity 2, and -1 as a zero of multiplicity 1.

- **Nonreal Zeros:** If a complex number $a + bi$, $b \neq 0$, is a zero of a polynomial function P with real coefficients, then its conjugate, $a - bi$, is also a zero. Hence, for a polynomial with real coefficients, nonreal zeros occur in conjugate pairs.

- **Irrational Zeros:** If $a + c\sqrt{b}$, where a, b, and c are rational and b is not a perfect square is a zero of a polynomial function P with rational coefficients, the its conjugate, $a - c\sqrt{b}$, is also a zero. Hence, for a polynomial with rational coefficients, irrational zeros occur in conjugate pairs.

Example 2: Find a polynomial of lowest degree with rational coefficients that has $-2i$ and $1 + \sqrt{3}$ as some of its zeros.

- **Rational Zeros Theorem**: If the polynomial

  $$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

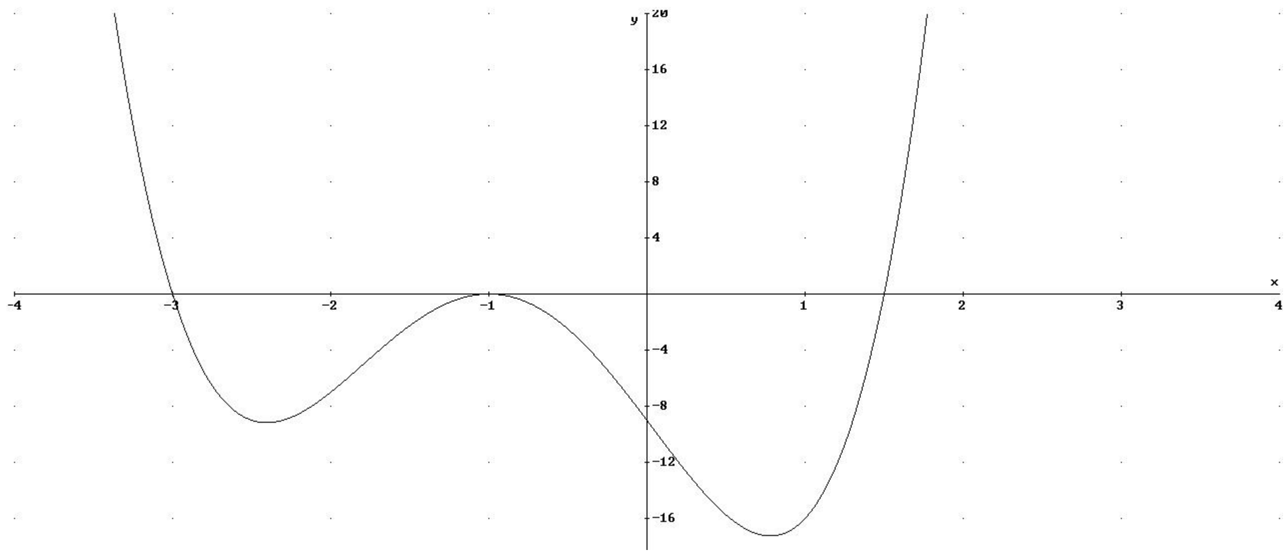
  has integer coefficients, then every rational zero of P is of the form $\dfrac{p}{q}$ where

  $$\begin{aligned} & p \;\text{ is a factor of the constant term }\; a_0, \\ \text{and } & q \;\text{ is a factor of the leading coefficient }\; a_n. \end{aligned}$$

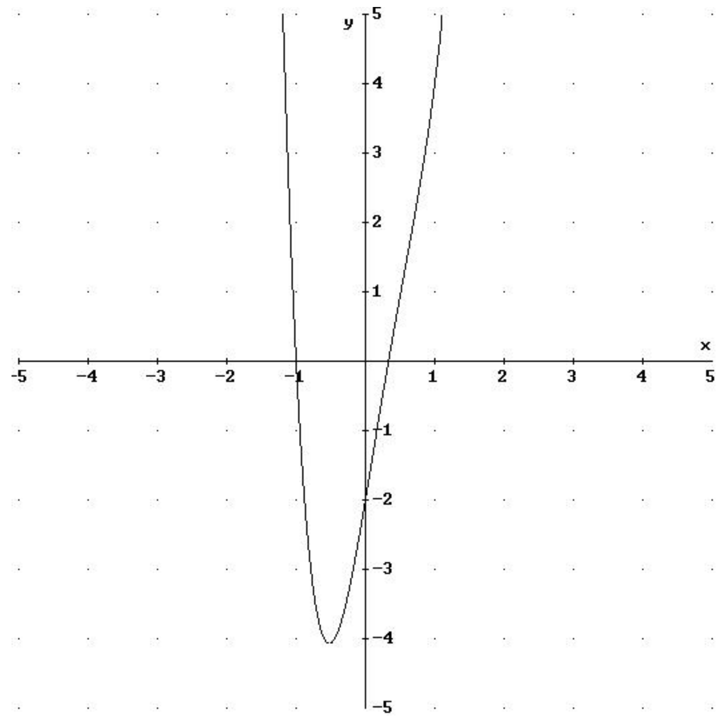
- The Rational Zeros Theorem does NOT list irrational zeros. These will need to be found using other means like the quadratic formula.

Example 3: List all POSSIBLE rational zeros of $P(x) = 2x^3 + 15x^2 + 22x - 15$.

Example 4: For $P(x) = 2x^4 + 7x^3 - x^2 - 15x - 9$, find all zeros of P and then factor P into linear factors.



Example 5: For $P(x) = 3x^4 - 4x^3 + x^2 + 6x - 2$, find all zeros of P.



Example 6: For $f(x) = 2x^6 - 3x^5 - 13x^4 + 29x^3 - 27x^2 + 32x - 12$